RESEARCH PROPOSAL

RHODES UNIVERSITY

FACULTY OF EDUCATION

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Student no	**: 13K6743**
Degree	**: MEd (full thesis)**
Department	**: Education**
Field of Research	**: Mathematics Education**
Supervisor	**: Prof. Marc Schäfer**

Provisional Title: An analysis of selected grade 11 learners' interactions with geometry tasks using visualization processes.

ABSTRACT

This Namibian case study aims to explore visualisation processes that eight grade 11 learners use in their interactions with selected geometry tasks. The eight participants will engage with 12 items of the Visualisation Geometry Tasks (GVT) worksheet. They will be video recorded as they solve each task and interact with me as they talk through their solution strategies. The emphasis will be on how they use visualisation processes to solve the individual tasks. The significance of this study stems from the awareness amongst mathematics education researchers of the role of visualisation in the teaching and learning of mathematics.

The Namibia Senior Secondary Certificate Ordinary (NSSCO) Examiners report (2014) states that geometry is one of the topics that proved to be difficult for learners, and thus recommend specifically that teachers must concentrate on this topic (p. 311). They also say learners must be taught to show complete methods of their working when answering geometry problems. To this end, this study aims to inquire into the nature of how visualisation in teaching mathematics can facilitate the learners' solving of geometry problems.

COMMON STATEMENT

This proposed research study is part of the "Visualisation in Namibia and Zambia" (VISNAMZA) project which seeks to research the effective use of visualisation processes in the mathematics classroom in Namibia and Zambia (Schafer, 2015). Research in the VISNAMZA project is currently centred around 5 MEd studies and 1 PhD study.

FIELD OF RESEARCH

Mathematics education: Visualization processes in Geometry

PROVISIONAL TITLE

An analysis of selected grade 11 learners' interactions with geometry tasks using visualization processes.

CONTEXT

It is well known that the education reform project in Namibia began in 1990 (National Curriculum for Basic Education, 2010). The primary goals for this reform were identified as ensuring access, equity, quality, and democracy in education (Ministry of Education [MoE], 1993). One of the vehicles for achieving the educational goals was the adoption of learner-centred education (LCE) (Namibia: MoE, 2003). The National Curriculum for Basic Education (NCBE) (Namibia: MoE, 2010a,) suggests that learners learn best when they are actively involved in the learning process through a high degree of participation, contribution and production. The document further outlines a variety of classroom techniques that a teacher should use. These techniques include direct questioning, eliciting responses, explaining, demonstrating, challenging the learners' ideas, checking for understanding, helping and supporting, providing for active practice, and encouraging problem solving (p. 26)

The LCE approach to teaching and learning according to the curriculum also encompasses "a text-rich and visually and tactile-rich learning environment" (Namibia: MoE, 2010a). The national curriculum further outlines that knowledge production is "shared through displays of learners' work, charts, posters, and easily accessible information sources" (p. 27). This approach emphasizes the value and use of visualization in teaching and learning of mathematics. In addition to the call for the use of visuals in teaching by the NCBE, the Namibian Senior Secondary Mathematics syllabus (Namibia: MoE, 2010b) states that one of its aims for all learners "is to help them recognize when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem" (p. 2).

Makina (2010) regards visualization “as a very important cornerstone in ‘teaching for understanding’ in mathematics because it helps the teacher with facilitation of lessons and with the ability to engage learners in realistic situations”. In an effort to gain insights into the affordances potentially offered by employing visualization in learning, the following key constructs of my study are discussed: visualization and geometry. The next section explores visualisation in mathematics teaching as the first construct of this study.

VISUALISATION IN MATHEMATICS

Visualization is recognized as an important aspect of mathematical reasoning (Elliott, 1998, p. 45). Visualization plays a role not only in geometry but in other domains of mathematics, such as algebra, transformation and number sense. By way of example: in algebra, imagining and drawing the curve of an algebraic equation $y = 10 + 2x – x^2$ is fundamental to understanding the curve; in transformation, imagining and drawing a transformed object like a right-angled triangle in a given vector, helps learners to understand translation beyond the abstract algorithmic procedures; in number sense, the imagining and drawing of number lines when adding or subtracting integers helps learners to build visual structures to interpret and understand addition and subtraction.

As to why visualization plays an important role in mathematics, there are a range of reasons: Wheatley and Brown (1994) state that 'activities encouraging the construction of images can greatly enhance mathematics learning'. Zimmermann and Cunningham (1991) argue that mathematical visualization is the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding (p. 3). They maintain that visualization supplies depth and meaning to understanding, serving as a reliable guide to problem solving, and inspiring creative discoveries. Van de Walle, Karp and Bay-Williams (2014,) propose that “visualization might be called geometry done with the mind’s eye” (p. 452). They further state that it involves “being able to create mental images of shapes and then turn them around mentally, thinking about how they look from different viewpoints-predicting the results of various transformations”. Anderson-Pence, Moyer-Packenharm, Westenskow, Shumway and Jordan (2014) argue that “visual representations alleviate cognitive load during problem solving and allow learners to mentally work on one part of the model without having to keep track of the entire model in their minds" (p. 3).

Visualization plays different functions or roles as students use it to solve different mathematical tasks. According to Ho (2010), the functions of visualization are to:

> understand the problem, simplify the problem, help learners see connections to a related problem, allow individual learning styles, can be a substitute for computation, can be a tool to check for solution and lastly plays a function of transforming the problem into a mathematical form" (p. 3).

Furthermore, visualisation helps learners to demonstrate their reasoning when given a mathematical task through the use of diagrams or mental processes.

Serpil, Cihan, Sabri and Ahmet (2002) reiterates that using a visualization approach in teaching and learning, many mathematical concepts in general and geometric concepts in particular can become concrete and clear for students to understand (p. 2). They add that visualization approaches can enable students to look at mathematics, which is usually seen as an accumulation of abstract structures and concepts from a different perspective (p. 3). Yakimanskaya (1980) states that visualization is essential to problem solving and spatial reasoning as it enables people to use concrete means to grapple with abstract images (p. 55).

In my view, visualisation can enhance mathematical understanding only if it is employed appropriately and effectively in a context of connecting with different mathematical domains. For these reasons, this study aims to first gain insight into the nature and how learners use visualisation processes in their thinking of mathematical tasks.

For the purposes of this study visualization is a key process in solving problems and making sense of mathematics (Zimmermann and Cunningham, 1991, p. 3). An important aspect of this study is the notion that visualisation is not an end in itself but a means towards an end.

GEOMETRY TEACHING AND LEARNING

Jones (2013) states that "one component of mathematics education that makes great use of diagrams is the teaching and learning of geometry" (p. 37).

A cornerstone of geometry is the visual study of shapes, sizes, patterns, and positions. In addition, Van de Walle et al. (2014) states that teaching and learning of geometry is based on two related frameworks: (1) spatial sense and geometric reasoning; and (2) the specific geometric content found in distinct objects (p. 427). They further assert that "the first framework has to do with the way students think and reason about shape and space while the second framework is content in the more traditional sense – knowing about symmetry, triangles, parallel lines, and so forth" (p. 426). Solving geometrical tasks in the context of this study refers to the meaningful solution strategies that learners show in obtaining their geometry answers.

Olkun, Sinoplu and Deryakula (2005) emphasise that improving students' geometric thinking is one of the major aims of mathematics education since geometric thinking is inherent in so many scientific, technical and occupational areas as well as in mathematics itself (p. 1). Visualization is a key component in geometric thinking as it enables students to represent abstract concepts in a visual manner.

For this reason, my study focuses on visualization in geometry learning. According to the theory of Pierre and Dina Van Hiele (1986), students progress through five levels of thought in geometry (p. 27). Their theory is based on several assumptions which assert that learning is a continuous process characterized by a journey through qualitatively different levels of thinking. These levels progress from "holistic visual thinking to analytical thinking to rigorous mathematical deduction" (p. 27). Clements (2003) argues that if a student is at the first van Hiele level, he or she recognizes shapes as wholes but cannot form mental images of the individual components of those shapes. At the second level, the student thinks in terms of the properties of the shape but may not know which ones are necessary to determine the shape. In the third level, students can form definitions with appropriate and sufficient sets of conditions for a geometric concept. They can also provide logical arguments in describing geometrical relationships (p. 152). A number of research studies (Schafer & Atebe, 2013; Atebe 2013) confirm Van Hiele's assertion that students who bypass the individual van Hiele levels find it very difficult to fully understand the geometry content that is being taught in the subsequent levels.

In an effort to understand why learners struggle with solving geometry tasks, Giaquinto (2007) postulates that ‘reliable justifications of belief can come from direct visual appraisal’. He explains this as follows:

> Our initial geometrical conceptualisation of basic shapes depends on the way we perceive those shapes. In having geometrical concepts for the shapes, we have certain beliefs forming dispositions. These dispositions can be triggered by experiences of seeing or visual imagining, and when that happens we acquire geometrical beliefs. The beliefs acquired in this way constitute knowledge, in fact synthetic a priori knowledge, provided that the belief-forming dispositions are reliable (p. 12).

Zodik and Zaslavky (2007) argue that geometric problems are often accompanied by figures/diagrams that represent specific mathematical ideas but a diagram can be accurate, sketchy or even misleading (p. 265). But also Acarvi (2003) argues that “in different contexts, the ‘same’ visual objects may have different meaning even to the experts” (p. 232). Employing visual support materials such as diagrams and sketches in the teaching and learning of geometry can assist learners to solve geometrical problems, present the solutions clearly and, check and interpret their results. Therefore, it is important that teachers use visual support materials strategically and correctly to tech geometry effectively.

According to Mateya (2013), Namibia recognizes mathematics as one of the crucial subjects necessary to realize the country’s full potential (p. 49). In the mathematics curriculum, “geometry” as a domain of mathematics is considered as a key element which should be taught proficiently (Mateya, 2013, p. 50). However, geometry which advocates for the use of visualisation in teaching and learning appears to be a problem mathematical domain in National examinations. According to the Namibian Senior Secondary Certificate Ordinary (NSSCO) Examiners report (2014), the question on geometry was moderately answered because some learners were unaware of circle geometry (p. 314). In addition, the NSSCO Examiners report for the past academic years (NSSCO, 2011, NSSCO, 2012, NSSCO, 2013) repeatedly encourage learners to show complete methods of working especially if they are asked to prove mathematical results. In an effort to achieve this it is important to encourage learners to use visualisation tools such as diagrams to illustrate their thinking and understanding (Presmeg & Nenduradu, 2005, p. 106).

My experience as a learner and a mathematics teacher for several years indicates that although learners at grade 11 level are generally familiar with some of the geometric shapes, they can hardly perform informal deductions let alone articulate a formal geometry proof. Learners make scant use of any kind of visualization processes in their engagement with geometric tasks. I argue this point because most learners prefer writing answers without showing how they obtain such answers.

Visualisation processes

For the purpose of this study, visualization processes are strategically selected visual methods, techniques and approaches adopted by the participating learners for specific mathematical task solving. These visual methods can include drawing visible images or forming mental images that might help the learners in solving a mathematical task. Yilmaz, Argun and Keskin (2009) conclude that "visualization is an important aspect of mathematical understanding, insight and reasoning" (p. 131). Although visualization processes are approaches that both teachers and learners employ in solving a given mathematical task, the focus of my study is on learners only.

This study aligns strongly with Ho's (2010) assertion that when students are presented with a mathematical task, they go through the following processes:

- **Understanding** the special relations of the elements in the problem;
- **Connecting** to a previously solved problem;
- **Constructing a visual representation** in the mind, on paper, or through the use of technological tools;
- **Using the visual representation** to solve the problem;
- **Encoding** the answer to the problem.

I will use Ho's analytic tool as a primary basis of my analysis for this study, which is discussed more fully below under data analysis.

THEORETICAL CONSIDERATIONS

This research study locates its theoretical underpinnings in individual and interactive constructions and re-constructions of knowledge. Ndlovu (2013) states that constructivism 'focuses on the internal, cognitive or conceptual development of the learner's mind or discipline [mathematics] as a whole' (p. 4). Similarly, Cobb (1988) reiterates that the fundamental goal of mathematics is or should be to help students build structures that are more complex, powerful, and abstract than those that they possess when instruction commences (p. 89). He further adds that teachers should devise teaching strategies that inevitably lead students to make appropriate and meaningful constructions. These strategies may include the use of drawings, diagrams and mental images. As learners draw or mentally form images (visualisation), they construct their own ideas and concepts (constructivism).

Constructivism and visualisation are linked and intersect at developing conceptual understanding for learners. As learners construct diagrams, pictures and shapes whether on paper or in their mind, conceptual understanding is enhanced. I argue this because both visualisation and constructing knowledge are intrinsically linked.

SIGNIFICANCE OF THE STUDY

The importance of visualisation in mathematics teaching and learning has been emphasised by many writers (Gorgorio & Jones, 1996, Melissa Rood, 2010, Rosken & Rolka, 2006). Further, the Namibian mathematics curriculum for the senior secondary phase expects "learners to use mathematics language and representation as a means of solving problems relevant to everyday life and to their further education and future careers on completion of the phase (Namibia: MoE, 2010a, p. 23). The examiners of the Grade 12 national examinations consistently identify geometry as an under-performed mathematical domain in these examinations (NSSCO, 2014, NSSCO, 2013, NSSCO, 2012). From my own experience, this leads me to speculate that perhaps the lack of visualisation practices and skills of the learners may be a contributing factor to this poor performance, hence the emphasis in this study on visualisation in geometry. Another reason for researching visualization processes in the Namibian context is that relatively little is known about these processes in Namibian learners.

In light of the reform that is taking place in the Namibian education system, teachers, curriculum designers, researchers and policy makers will gain insightful information about the importance of integrating visualisation processes in the learning and teaching of mathematics. Furthermore, this study will hopefully help and inform Namibia's teacher training institutions and curriculum designers to adequately incorporate visual processes into their curriculum reviews in order to foreground its fundamental importance. Moreover, this study is part of the VISNAMZA suite of studies looking at visualisation in mathematics in Namibia and Zambia, and will make an individual contribution to the collective aims of the project.

AIMS AND OBJECTIVES

The aims and objectives of this study are twofold: It is to analyse the **nature** of visualization processes employed when selected grade 11 learners interact with geometry problems, on the one hand, and to determine **how** these grade 11 learners use visualisation processes in their interactions with geometry problems on the other.

My fundamental research questions that will guide my research are:

- What is the nature of the visualization processes employed when selected grade 11 learners interact with geometry problems?
- How do these grade 11 learners use visualization processes in their interactions with geometry problems?

METHODOLOGY

Creswell (2006) defines a methodology as a philosophical framework and the fundamental assumptions of research (p. 4). He further argues that the philosophical framework one uses influences the procedures of research – the research methodology relates intimately to the entire project.

Orientation and method of the study

This case study research takes the form of a mixed method (Qualitative & Quantitative), interpretive case study. It is a case study because "it provides unique examples of real people in real situations" (Cohen, Manion & Morison, 2007, p. 253). It is interpretive because it recognizes that "individual' assumptions and experiences contribute to the on-going construction of reality" (Wahyuni, 2012, p. 71).

The mixed method approach will help me to analyse my data from different perspectives. This means data analysed qualitatively will complement the data analysed quantitatively. This adds to the validity and reliability of my study. Qualitative methods will be used to analyse how learners interact and engage with the Geometry Visualization Task (GVT) activities. A quantitative analysis using descriptive statistics will provide me with a broad overview of how participants employed visualization processes in their engagement with the GVT.

The qualitative analysis will provide me with rich, detailed understanding of the character of the visualisation approaches used, and why participants employed or did not employ certain visualisation processes in their engagement with the GVT.

The case in this study is a group of grade 11 learners interacting with a set of geometry tasks. The unit of analysis associated with this case is specifically their engagement with the GVT tasks with reference to the visualization processes they employ

Selection of Participants

The site where this case study will be conducted is conveniently sampled to allow the researcher easy access. This research will be conducted in the Zambezi Region of Namibia with eight learners selected from three grade 11 classrooms. The sampling of participants will be purposive. Cohen et al. (2011) states that "in purposive sampling, often a feature of qualitative research, researchers hand-pick the cases to be included in the sample on the basis of their judgement of their typicality or possession of the particular characteristics being sought" (p. 156). The reason for using purposive sampling is to focus on specific, unique issues or cases or sample characteristics and to generate rich data on these. Another reason for using this purposive sample is to allow me to have ample time working with these learners that I know well and I believe will be willing to give me the needed time.

Furthermore, the selection criterion for the participants will be performance-based. My intention is to work with a range of learners performing at different levels: below average, at average and above average in their mathematics lessons. The reason for using this performance-based selection is to seek to get insights into how learners across the performance spectrum use visualization processes when interacting with geometry tasks. An even number of male and female participants will be considered in order to ensure a gender balanced sample.

Geometry visualization tasks (GVT)

A set of geometry visualization tasks (GVT) will be designed into an instrument containing twelve different geometry tasks. Each task will take the form of an open-ended question in the sense that many solution strategies can be employed to solve each task. According to Brahier (2001), open-ended questions require far more careful assessment, as there is usually a variety of ways to answer the questions, but they have the potential to more accurately assess student thinking processes than the closed version of the question (p. 19). My interest in the solution strategies that the participants employ is to specifically identify the visualisation processes that they use as they work through each of the tasks. Each GVT question will have adequate space for the learners' writing and drawings. I will design the tasks to align them with the Namibian education context and curriculum.

Geometry visualization tasks – the GVT

Below are three examples that will be included in the final GVT:

1. A rhombus BDEF is inscribed in a triangle ABC. Its diagonal BE is perpendicular to the side AC of the triangle. Prove that triangle ABC is an isosceles triangle.
2. A man was searching for his cattle. He walked 3 km due north. Then he turned and walked roughly south – east for 5km. There he found his cattle and realized that he can get home by walking directly west.
   (a) What kind of triangle did the man walk?
   (b) How far did he walk to reach home after he found his cattle?
3. You can place exactly 9 and 16 small squares in the two squares formed on the other two shorter sides of a triangle respectively. How many small squares can you place in the square formed by the third side?

RESEARCH METHODS

DATA COLLECTION

For the purposes of this research study, observations and interviews will be used to generate data:

Observations

The observations consist of individual videos of the participants interacting with each task of the GVT and me. I will specifically be looking for data on the nature of the visualization processes learners employ as they interact with each task of the GVT. These processes will be identified and coded (as per analysis template discussed below) as each of the participants interacts with each of the twelve tasks.

Interviews

Two types of interviews will be used, namely, GVT stimulated recall-interviews and post GVT structured interviews:

GVT Stimulated recall-interviews

Using the GVT worksheet as a stimulus, I will observe and video record how each participant engages with each task of the GVT. This approach will allow participants to 'talk through' their thought processes as they interact with the GVT and me which in turn affords them the opportunity to justify how they solve each GVT task. In this type of interview, each participant will respond to GVT tasks and converse with me about how they solved the task. In particular I will encourage the participants to explain how they used their visualisation processes to solve the tasks. "This open-endedness allows the participants to contribute as much detailed information as they desire and it also allows the researcher to ask probing questions as a means of follow-up" (Turner, 2010, p. 756). The choice to use this technique is seek insight of how participants use visualisation in solving geometry tasks. The GVT stimulated recall interviews will be video recorded and then transcribed.

Post GVT interviews

I will arrange specific time slots to conduct an interview with each participant after their interaction with the GVT. The aim of this interview is for the participants to reflect on the GVT process and for me to ask follow-up questions and seek clarity on issues and ambiguities that may have arisen in the GVT interactions above. Data collected here will be transcribed.

RESEARCH PHASES

Data for this research will be collected in four phases:

Phase 1- Consent and pilot

In this phase, I will request permission from the Regional Director of Education and from the school principal to conduct this research study at the identified site. Furthermore, I will also seek permission from parents and from the selected participants respectively. In addition, the final crafting and piloting of the GVT will also take place in this phase.

Phase 2- GVT interactions

During this phase, the selected participants will interact with the 12 items of the GVT. The GVT will take the form of worksheets and interviews. Each task will have sufficient space for written and drawn solutions and will be analysed based on my analysis template discussed below. The participants will interact with the GVT each at different time after regular academic teaching hours using pencils, erasers and papers. I will interact with each participant as he / she works through the GVT in the worksheet and interview him / her against each question and have them "talk through" their thought processes. Participants will not be set any time limit to solve the tasks. The emphasis is on providing evidence of the visualisation processes they use to solve the tasks. In this way, I will gain insights of the visualization processes that the learners employed.

In order to capture data obtained from the above process, I will video record the participants interacting with the GVT and me for each task during the whole process. The analysis of this will answer research question number one.

Phase 3- Post GVT interview

In this phase, I will conduct a one-on-one interview with each participant after he / she completes the GVT. This will take place after I have completed an initial analysis round of the video recording of each participant from phase 2. Participants will reflect and elaborate on how each GVT task was solved using visualisation processes. I will also use this interview to clarify uncertainties that may have arisen in Phase 2.

Phase 4- Analysis

During this phase I will analyse the data.

DATA ANALYSIS

Data Management

The data which I will collect shall be arranged into labelled files for the purpose of accessing it. In addition, my work will be backed up electronically and saved in different locations such as personal email accounts to safe guard against loss of work.

Data Analysis

Cohen et al. (2011) claims that "in abiding by the principle of fitness for purpose, the researcher must be clear what he / she wants the data analysis to do as this will determine the kind of analysis that is undertaken (p. 538).

A template of observable indicators adapted from Ho (2010) will assist me to identify and classify visualization processes evident in the participants' interaction with the GVT and me. This template also contains classifications of representations adapted from Ho, Ramful and Lowrie (2014). Each task that each participant will solve shall be subjected to this analysis. The indicators are:

1. **Understanding the problem.** This aspect will be demonstrated by learners if they are able to correctly represent the given task visually. For example, if learners are asked to calculate the sum of the interior angles in a pentagon, are learners able to draw the correct shape and identify the appropriate angles inside the pentagon?
2. **Connecting to a previously solved problem.** Visualization allows students to relate the current problem to previous ones and identify a simpler version of the problem and method that works for the set problem. For example, in establishing the interior angles of a polygon one traditionally uses the result that the sum of the interior angles of a regular polygon is $(n - 2)\ 180^0$ where *n* is the number of sides of the polygon. This formula allows learners to simply substitute the number of sides given. In my study I wish to see how the participants use a visualisation process of this strategy. The observable indicator

will thus be seeing learners dividing the given polygon into triangles that they know have sums of interior angles of 180^0.

3. **Constructing a visual representation.** Here I will be specifically looking for any visuals that the participants construct when solving the tasks of the GVT. These can include diagrams, scribble or even images in the mind expressed in words.
4. **Using the visual representation to solve the problem.** Here I will be looking specifically for how the visual representation crafted by the participant is used in solving the given task. Can the answer to the given task be obtained directly from the visual representation itself, without the need for computation, or is the visual used as a scaffolding mechanism to solve the task at hand?
5. **Encoding the answer to the problem.** Here I will observe the link between the solution and the visual representation crafted by the participants. This indicator will help me measure the reasonableness of the answers in relation to the visuals presented.

I intend to use the template illustrated in Table 1 to code the above indicators as I observe them in action and in the videos for each participant. To complement the findings on the table, a descriptive statistic such as frequency tables and bar charts will be used to illustrate the trend of the data.

Table 1: Analytical instrument - Table of indicators to identify and classify visualization processes for each task of the GVT for each participant.

Number	**Indicator of Visualization** The visualization process/sketch assists in:	**Accurate visual-schematic representation**	**Inaccurate visual-schematic representation**	**Accurate pictorial representation**	**No visual representation**
1	Understanding the special relations/properties inherent in the geometry task at hand				
2	Making connections to previous geometry tasks encountered				
3	Transforms the task into a mathematical form				
4	Clarifying the task at hand				
5	Illustrating the problem scenario				

Source: indicators adapted from Ho (2010) and Ho, Ramful and Lowrie's (2015) classification of representations to analyse the data collected in the study.

SUMMARY TABLE OF THE RESEARCH PROCESS

Phases	Instrument	Purpose	Data	Analysis
1	Securing participants and seeking consent. Designing and piloting of GVT	Ethical implications Develop the instrument to ensure validity Testing the GVT's efficacy	N/A	N/A
2	Participants interacting with GVT	Generate data Video recording of interactions	Interactions of learners with GVT	Quantitative and qualitative
3	Post GVT interview Participants answer research question two.	Validate findings through comparability of generated data	Visual representations in form of diagrams to questions in GVT	Qualitative
4	Analysis of data	Obtain insight into how learner use visualisation processes in solving geometry problems.	Video data verbal responses, drawings and diagrams	Quantitative and qualitative

VALIDITY

According to Cohen et al. (2001), validity is "an important key to effective research" (p. 106). Maxwell (1996) states that the readers and users of this research report expect assurances that the data and research findings are valid and reliable and not skewed by the researcher's own perspectives and ideologies. In an effort to address the issues of validity and reliability, I will use a variety of data generation methods to triangulate the data. This data shall be interpreted based on underpinning theory and analytical frameworks following the case study protocol and not my own views and opinions.

Audio recordings and written interview transcriptions will be analysed in iterations and cycles. Furthermore, member checking will be employed by circulating the transcripts and my data summaries to the research participants for confirmation that they are accurate reflections of what they said. I also intend to pilot the GVT with five grade 11 learners for ambiguous language and any other inaccuracies.

ETHICS

See attached form of RU Faculty of Education: Ethical approval application.

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RU FACULTY OF EDUCATION: ETHICAL APPROVAL APPLICATION

IMPORTANT: The following form needs to be completed by the researcher and submitted with their research proposal to the Education Higher Degrees Committee. The details to which this form relates should also be evident in the text of the proposal.

GENERAL PARTICULARS

MEd (Half thesis)		**MEd (Full thesis)**	×	**PhD**		**Other: Please specify**	

TITLE OF RESEARCH: An analysis of selected grade 11 learners' interactions with geometry tasks using visualisation processes.

DEPARTMENT/INSTITUTE: Education Department

DATE: [Submission to EHDC]: December 2015

RESEARCHER: Brian Simasiku Kabuku

SUPERVISOR: Professor Marc Schafer

ETHICS

NB: You must read the Faculty of Education Ethics Guideline ***prior*** to completing this form.

Please indicate below how your research supports the indicated ethical principle:

Respect and dignity

I will, from the beginning represent the study as a project that participants feel part of through two-way communication. I will explain the objective of the project honestly to gain their trust and their contribution through their participation will be explicitly valued. Participants' anonymity will be ensured and their participation will be entirely voluntary. This means participants are free to withdraw at any time without any consequences attached. Furthermore, I will code the identities of the participants to ensure confidentiality of the data. In addition, I plan to be working on this project after school academic hours to avoid engaging learners in the project during teaching time.

Transparency and honesty

To ensure transparency, informed consent in writing from district officials, the principal, participants and parents will be sought. The participants will be involved in every phase of the project by affording them the opportunity to verify their responses in the stimulated recall-interview. On-going conversations between participants and I during and after the Geometry Visualisation Tasks (GVT) interactions will also provide a sense of transparency and honesty to the participants of the project.

Accountability and responsibility

The power relationship issue between participants and I which might influence the outcome of the research shall be considered to avoid leading participants into giving responses that are not entirely honest. To overcome power related biasness and issues, I will establish a positive rapport between the participants and myself. To a large extent this has already been established as the community I work in is relatively small.

Integrity, academic professionalism and researcher positionality

In order to conform to the standard of academic professionalism, I will use multiple data collection techniques which encompasses observations, follow-up interviews and the GVT to ensure that data collected is authentic. The findings of this study will be documented whether they are in conflict with my initial expectations and assumptions or not. Furthermore, the final product of this project shall be made available to the public (teachers, curriculum designers and policy implementers) to serve its intended purpose of sharing the findings on the significance of using visualisation in mathematical learning. I am aware that my participants are minors and will thus engage with them with integrity at all times upholding all ethical standards of Rhodes University. I will acknowledge that the thesis is my own work and I will adhere to the protocols of the Rhodes University Referencing Guide to reference the work of others appropriately.

Signature (researcher): Brian S Kabuku Signature (supervisor):Marc Schafer

Date: 14 October 2015 Place: NIED Okahandja