RHODES UNIVERSITY
FUCULTY OF EDUCATION
RESEARCH PROPOSAL

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Degree	: Master of Education (Full thesis)
Department	: Education
Field of Research	: Mathematics Education
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An analysis of how GeoGebra can be used as a visualization tool by selected teachers to develop conceptual understanding of the properties of geometric shapes in Grade 9 learners.

Abstract

According to Rosken & Rolka (2006) learning mathematics through visualisations can be a powerful tool to explore mathematical problems and give meaning to mathematical concepts and relationships between them. "Visualisation can reduce the complexity of mathematical problems when dealing with multitude of information" (p.458). This research proposes to study the role of visualization in the teaching and learning of angle properties of geometric shapes in grade 9. The intervention at the heart of this research uses GeoGebra visualisations to teach angle properties in Grade 9 Geometry. The study analyses the role of GeoGebra visualisations in teaching and how it could enhance conceptual understanding.

Common statement:

This proposed research study is part of the "Visualisation in Namibia and Zambia" (VISNAMZA) project which seeks to research the effective use of visualisation processes in the mathematics classroom in Namibia and Zambia (Schäfer, 2014). Research in the VISNAMZA project is currently centred around 5 MEd studies and 1 PhD study.

Field of research: Visualisation in mathematics education

Provision title: An analysis of how GeoGebra can be used as a visualization tool by selected teachers to develop conceptual understanding of the properties of geometric shapes in Grade 9 learners.

Context

Introduction

The broader Namibian curriculum for basic education advocates that a stimulating learning environment is a text-rich and a visually and tactile-rich learning environment. The curriculum further states that, "Effective learning and teaching are closely linked to the use of teaching and learning materials (e.g. books, posters, charts or recycled waste materials, etc.) and ICTs (e.g. computers, audio and visual media) in the classroom"(Namibia. Ministry of Basic Education [MBE], 2010, P.27). Similarly, Bishop (2003) in his review of research on visualisation in mathematics concludes that there is value in emphasising visual representations in all aspects of a mathematics classroom. He explains that mathematics is a subject that is concerned with the study of patterns, representations and sets of connected ideas. Many of these representations appear to be visual, having roots in visual sensed experiences.

As a Junior Secondary teacher for mathematics for over 14 years, I have observed learners struggling to understand geometric terminologies and concepts. In particular, Grades 9 and 10 learners find it difficult to distinguish between corresponding angles, co-interior angles, alternate angles and vertically opposite angles formed within parallel lines. I also noted that learners often misunderstand, or are unaware of the properties of angles in triangles and quadrilaterals. Being passionate about mathematics and technology, I have always used GeoGebra as a visualisation tool to draw clear and accurate diagrams for worksheets or test papers. Since GeoGebra can also display dynamic diagrams I am convinced that it can be used by teachers in their teaching as a powerful visualisation tool to explain mathematical concepts more effectively.

Arcavi (2003) suggests that "visualisation is no longer related to the illustrative purposes only, but is also being recognised as a key component of reasoning (deeply engaging with the conceptual and not the merely perceptual), problem solving, and even proving" (p.235). He proposed that visualisation offers an opportunity of seeing the unseen, which he referred to as visual imagery. Visual imagery is the ability to form mental representations of objects and manipulating them in the mind (Presmeg, 1985 and Koysslyn, 1994). Presmeg (1992) articulates that visualisation is an aid to understanding and visualising a mathematical concept or a problem refers to forming a mental image of the problem. To this end the following three key concepts in this study, ie **visualization**, **GeoGebra** and **conceptual understanding** are now discussed.

Visualisation

What is visualization?

According to Arcavi (2003, p.217) visualization is:

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images diagrams, in our minds on a paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding.

He views visualisation as a powerful tool that plays three major roles in the learning and teaching of mathematics: Firstly visualisation can support and illustrate essential symbolic results and possibly provide proof in its own right. Secondly, visualisation can provide a way of resolving conflict between correct symbolic solutions and correct intuitions. Thirdly, visualisation can help learners to engage with concepts and meanings on a level that is not only symbolic and abstract.

Hershkowitz (1989), Zimmerman & Cunningham (1991) emphasised both the physical and mental aspects of the visualisation process. They describe mathematical visualisation as the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding.

Mesaroš (2012) suggests that the primary aim of visualization in teaching mathematics is to facilitate and support the pupil's solving process (ibid). He further says that visualization helps in transforming a mathematical problem into a form of an image. This image enables the solver to better understand problems whose solution would otherwise be inaccessible without using visualisation.

Of particular relevance to this study, Card, Mackinlay and Shneiderman (1999) emphasised that visualization, specifically by means of a computer can support the visual representation of abstract data. Computer visualization can enable the concrete visual representations of mathematical concepts. It can enhance these with additional features such as movement, interactivity and interconnecting multiple visual representations simultaneously. For example, the result that the sum of angles in any triangle is 180° can be dynamically visualised using GeoGebra, by sketching the triangle and then dragging one of the vertices to immediately create a new triangle and observing that the sum remains constant.

In his writing on learning with visualization, Van De Walle (2004) referred to visualisation as:

> 'Geometry done with the mind's eye'. It involves being able to create mental images of shapes and then turn them around mentally, thinking about how they look from different perspectives predicting the results of various transformations. It includes mental coordination of two or three dimensions predicting the unfolding of a box (or net) or understanding a two dimensional drawing of a three-dimensional shape. (p. 429)

Kosslyn (1994) describes visualization as a cognitive process that involves visual imagery which he also refers to as mental representations or pictures in the mind. According to Kosslyn, mental representations (visual imagery) are important because it facilitates visualization processes whereby images are generated, inspected, transformed or used for mathematical understanding. In his work, Kosslyn (1994) proposed four cognitive steps involved during visualization processes. These are *image generation, image inspection, image transformation* and *image use.*

- **Image generation** occurs when a person produces a picture in his/her mind. In this process the learner pictures him/herself in an activity in which he/she is doing the moving of pictures or images in his/her imagination..
- **Image inspection** involves examining an image in order to answer questions about it. It is therefore important for remembering shapes. Shapes during this process are recognized in terms of their properties. During image inspection learners are estimating sizes, creating, recognizing and naming shapes. This process allows learners to connect visual images and abstract conceptualizations by seeing, looking for and describing patterns as basic forms of mathematical thinking.
- **Image transformation** is when one changes or operates upon an image, changing it into other related shapes. Dynamic software like GeoGebra can allow image transformation to be immediate and directly observable. Learners can observe the dynamic change of the picture like dragging a vertex of a square to form a kite, or holding and dragging a side of a square to form a rectangle or a parallelogram.

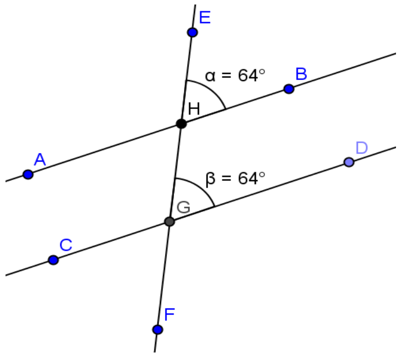
- **Image use** occurs when an image is employed in the service of some mental operation that includes comparing properties of images or answering questions about an image.

In relation to the above visualisation processes, Kosslyn (1994) suggests that these processes are hierarchical. A learner has to generate the image first, to be able to inspect it, transform it and then be able to use it. He gives a case where a learner generated an image and failed to inspect or describe it in a way that would aid him/her to solve the problem. In such cases there is a need to regenerate another image and take it through the processes above. Since the four visualisation processes identified by Kossylyn (1994) are applicable to visualisation in the mind, I will adapt this framework to visualisation processes that are observable as my participating teachers teach using GeoGebra as a visualisation tool.

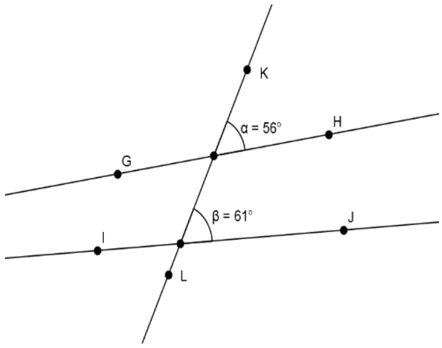
GeoGebra

For me to research the role of visualisation in mathematics, I choose GeoGebra as a visualisations tool over others because according to Bu & Schoen (2011) GeoGebra is particularly well suited for teachers to represent diagrams in different ways on the screen and dynamically transform them. Learners can gain from the use of this software, as they can observe and work with diagrams from different angles and perspectives on the computer screen. The diagrams can be moved around and manipulated in many ways. As a consequence, the learners are able to gain rich experiences from a variety of forms of the images that are different from the static diagrams in text books. The dynamic nature of the software offers exciting opportunities for learning and teaching mathematics in schools. According to Gerrit (2009), Hohenwarter, Hohenwarter and Lavicza, (2009), if used effectively it helps learners and teachers to specifically make connections between **Geo**metry and al**Gebra.** According to Kilpatrick, Swafford & Findell (2001) the ability to make mathematical connections is one of the key indicators of conceptual understanding. Hohenwarter & Jones (2007) emphasised that GeoGebra is very visual and dynamic as it was developed to enable multiple representations and visualization of mathematical concepts in a very dynamic manner. It is a free and open source software package which encourages teachers and learners to use it both within the classroom and at home. It combines dynamic geometry, algebra, calculus and spread sheet features into a single package. However the focus of this study is geometry only.

Using GeoGebra, teachers and learners can engage in a variety of exploratory activities such as drawing, constructing, testing, creating and manipulating any plane figure they desire to solve. The software is designed to generate very accurate diagrams and images. For example, a teacher wants to explain the behaviour of corresponding angles which are formed when a transversal crosses two lines. Corresponding angles are the ones at corresponding locations of the transversal line such as α and β in figure 1 below. In a text book or on a chalkboard these representations are static. The dynamic nature of GeoGebra is such that the sizes of angle α and β in figure 1 can be manipulated by re-orientating lines AB and CD to different positions. In the process the teacher can ask learners to immediately observe the behaviour of angle α and β. They then discover that once AB and CD are parallel the corresponding angles α and β are equal. Conversely they can also discover that GH and IJ are not parallel because the corresponding angles are not equal as shown in **figure 1**



Corresponding angles in -parallel lines are equal



Corresponding angles in non-parallel lines are not equal

Figure 1: shows the corresponding angles in parallel lines and in non-parallel lines

As illustrated in figure 1 above, GeoGebra allows the direct manipulation and reorientation of lines and points by its drag function. The movement produced by the drag function is a way to visualise the properties that define the figure when certain parameters of the lines are changed (Chiappini & Bottino, 1999). For this study I will use GeoGebra visualisations to analyse Kosslyn (1994) visualisation process of image generation, inspection, transformation and image use. See the analytical tool shown in table 1.

The notion that the use of GeoGebra visualisations encourage and enable teachers and learners to explore mathematical relationships and concepts in a dynamic manner aligns well with teaching for conceptual understanding.

Conceptual understanding

Kilpatrick et al., (2001, p.5) describe conceptual understanding as the comprehension of mathematical concepts, operations and relations and suggest that conceptual understanding involves the ability to integrate and connect mathematical ideas. These may be ideas about shapes and space, measures, patterns, functions, connections, proofs etc. With conceptual understanding “Learners gain confidence, which then provide a base from which they can move to another level of understanding” (pp.118-119).

Learners with conceptual understanding know more than isolated facts and methods. They understand why mathematical ideas are important and the contexts in which they can be used. Learners are able to organise their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Connections are most useful when linked to related concepts and methods in appropriate ways. According to Kilpatrick et al., (2001) one of the significant indicators of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. These different representations include different visualisation such as diagrams, computer images and others. Knowledge that has been learned with understanding provides the basis for generating new knowledge and solving new and unfamiliar problems. Kilpatrick et la., (2001) believe that, “when learners have acquired conceptual understanding in an area of mathematics, they see the connections between concepts and procedures and can give arguments to explain why some facts are consequences of others (p.119).

The following are key conceptual indicators, mostly adapted from Kilpatrick et al., (2011)

Connecting mathematics to prior-knowledge

This includes the ability to see connections between the mathematics that learners are learning and what they already know. This implies that learners should learn mathematics with understanding, actively building new knowledge from past experiences. This involves adapting acquired knowledge to new situations, and uses it to solve new mathematical problems.

Justifying and explaining mathematical ideas and solutions

This refers to the ability to provide evidence for clearly explaining and articulating mathematical concepts and ideas. Leaners are able to manipulate representations, compare concepts, and apply facts and definitions to justify solutions to mathematical problems.

Representing mathematical concepts in different ways

This indicator refers to learners’ abilities to show different representations of the same mathematical concepts. In this study different representations include different diagrammatic and visualization forms of a mathematical concept.

Connecting ideas and concepts in mathematics

This indicator refers to learners being able to discuss similarities and differences of representations and how they connect with each other. They are able to integrate related mathematical concepts and principles.

Connecting mathematics to real world

This indicator is about the ability of learners to connect and link mathematical knowledge to the outside world and seeing the practical relevance of this knowledge.

Theoretical considerations

The use of technology in the teaching and learning of mathematics offers an abundance of opportunities to make the classroom an interesting and inspiring space for learning. GeoGebra, if harnessed appropriately, is particularly well suited to facilitate a learning process that is interactive and activity-based. Using GeoGebra as a visualizations tool aligns well with constructivism as a theory of learning. This theory argues that humans generate knowledge and meaning from interaction between their experiences and their ideas (Piaget, 1967). Social constructivism emphasizes that this interaction occurs in a social context and is based on interpersonal relations. The theory claims that learners learn mathematics through active construction of their own knowledge, rather than receiving it as a finished product from the teacher or texts (Ernest, 1991). According to Vygotsky (1962) learners cannot be given knowledge, but instead they learn best when they discover things, build their own theories and try them out rather than simply consuming what they are told or instructed. Vygotsky argues that: “direct teaching of concepts is impossible and fruitless. A teacher trying to do this accomplishes nothing but empty verbalism, a parrot-like repletion of words by the child, simulating a knowledge of the corresponding concept but actually covering up a vacuum” (Vygotsky, 1962, p.83). Using interactive software encourages leaners to interact with the mathematical concepts in ways that are exploratory. It encourages learners to construct knowledge by active engagement. If used appropriately it can also be used in a social milieu that is interactive and collaborative. This is central to my study as I intend to use GeoGebra interactively in such a way that the learners explore mathematical concepts using the visualization potential of the software package. It is envisaged that the use of GeoGebra in my study will enable learners to use visualizations in different ways on the computer screen and transform them to make connections and discoveries. Through activities that are consistent with social constructivism, learners have the opportunity not only to learn mathematical skills and procedures, but also explain and justify their own thinking and discuss their observations (Silver, 1996). This is supported by Hyles (1991) who argues that mathematics lessons can be enhanced by using computer technology that encourages social interaction and collaboration. A key element of this study is the learners’ manipulation of GeoGebra images in combination with interacting with each other.

Significance of the study

Having taught mathematics for 14 years, I have observed that mathematics teaching practices in our classes have relatively little connection with actual mathematics. In my experience teachers teach mostly through rote learning of mathematical formulae and rules for solving mathematical problems. Teachers rarely use interesting teaching aids such as visuals, computers and charts to exemplify and describe mathematical ideas and concepts. This study seeks to challenge such practices in our mathematics classroom by using GeoGebra as a visualisation tool. "GeoGebra has been characterized by several authors to be a conceptual tool, a pedagogical tool, a cognitive tool, or a transformative tool in mathematics teaching and learning" (Bu & Schoen, 2011). Learners need to learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. GeoGebra has the potential to create visualisations which offer opportunities for learners to be actively involved in understanding mathematical concepts and explore mathematical ideas which will enhance their mathematical conceptual understanding. Teachers and policy makers who read this study will hopefully gain insight of how GeoGebra can be used as a powerful visualisation tool to enhance conceptual understanding.

GeoGebra software will allow learners to see mathematical concepts represented in different ways on the screen and transform them. Learners can gain from the use of the software, as they observe diagrams from different angles on the screen. As a consequence they will gain a rich experience that will allow them to form dynamic images to work with.

Goals

The proposed case study aims to firstly investigate the role of GeoGebra as a visualisation tool by observing selected teachers teaching Grade 9 learners using GeoGebra. Secondly this study analyses how these teachers use GeoGebra visualisations to enhance conceptual understanding of geometric angle properties.

Research questions

My research questions ask what selected teachers' perceptions and experiences are of:

1. The role that GeoGebra visualizations can play in developing conceptual understanding in the teaching of properties of shapes in Grade 9 geometry?
2. How GeoGebra can be used as a teaching tool to enhance learners' conceptual understanding of angle properties in Grade 9 geometry?

Methodology

Research Orientation

The proposed study is conducted within the interpretive paradigm. Kilpratrick,(1988, p 98), states that interpretivist research intends to, “capture and share the understanding that participants in an educational encounter have of what they are teaching and learning”. Cohen,Manion & Marrison, (2011, P.17) expressed this intention by saying that the “central endeavour in the context of the interpretive paradigm is to understand the subjective world of human experience”. Cohen et la., (2011) continue to emphasise that in order to retain the integrity of the phenomena being investigated efforts should be made to get inside the person and to understand from within. Interpretive researchers make interpretations with the purpose of understanding human agency, attitudes, beliefs and perceptions. In choosing this topic I am hoping that the co-participants teachers will be encouraged to share their experiences and perceptions pertaining to GeoGebra as a visualization tool. “The sense of data can only be drawn from the interaction between researcher and respondents” (Betram & Christiansen, 2014, p.16). My study seeks to specifically analyse selected teachers’ perceptions and experiences of the role of GeoGebra visualisations and how they are using the software to enhance conceptual understanding.

Methods

This research project is a qualitative case study. According to Miles & Huberman (1994), in qualitative research “the researcher attempts to capture data on the perceptions of local actors from the inside through a process of deep attentiveness of empathetic understanding and suspending perceptions about the topic under discussion” (p.6). Cohen et la., (2011), state that a case study “ provides a unique example of real people in real situation,” in that it enables readers to understand ideas more clearly rather than simply being presented with abstract theories, or ‘principles’ (p.289). Yin cited in (Cohen et la., 2011, p.290) echoed the same sentiment that “case studies have the advantage of including direct observation and interviews with participants”. This study will engage with 3 purposefully selected mathematics teachers. Two teachers and I will teach three grade 9 classes at my school using GeoGebra as a visualisation tool. The unit of analysis will be the perceptions of the participating teachers with regard to the role of GeoGebra visualisations and how they used the software to enhance the conceptual understanding in the learning of angle properties in grade 9 geometry.

Participants/Sample

Three teachers (two colleagues and I) have been purposefully selected to participate in this research project. Cohen et la., (2011) emphasised that in “purposive sampling, a researcher hand picks the participants to be included in the sample on the basis of their typicality or possession of the particular characteristics being sought" (p. 156). Purposive sampling thus enables me to select participants that are most suited for this research project, viz mathematics teachers who are interested in using GeoGebra in their teaching. Onwegbuzie and Leech (2007:249) noted that many times, the purpose of sampling is not to make generalisations, neither to make comparisons but to present unique cases that have their own intrinsic values. The proposed study aims to include mathematics teachers who are at an advanced level in computer skills. Therefore two teachers and I will be involved in this study. The two selected teachers were presenters in the Oshana regional e-learning conference in 2015. It is however important that the participants have a shared understanding of GeoGebra and visualisation – hence the training programme proposed for phase 1 below.

Research Design
My study is divided into five phases.

Phase 1 – Installation of GeoGebra and training of participants
In this phase I intend to install the GeoGebra software onto 30 laptops which are housed in the computer laboratory at my school. This phase also consists of a GeoGebra training programme for the two co-participants of this research project. This programme consists of 6 workshops where I will train the two colleagues how to use GeoGebra. Integral to the training programme is creating an awareness of conceptual understanding in mathematics and how GeoGebra can support the development of conceptual understanding. I will also make use of GeoGebra tutorial videos on YouTube to consolidate my input. At the end of the training programme, two workshops will be held to design four lessons. These lessons incorporate GeoGebra to teach angle properties as articulate in Phase 3. Each teacher will also plan and design a pilot lesson which he/she will implement in Phase 2.

Phase 2 – planning and piloting
During this phase I will introduce learners to use GeoGebra. All three grade 9 classes at my school will be trained on how to use this software. The Pilot Curriculum Guide for Formal Basic Education under the Ministry of Basic Education, Sport and Culture [MBESC], (1996) emphasises that "Learners learn best when they are actively involved in the learning process through a high degree of participation, contribution and production" (P.26). So the teaching method should be encouraging the active involvement and participation of learners. It is the intention of the intervention in Phase 3 for learners to be fully and actively involved in using GeoGebra, and not, as is often the case simply watching the teacher using the software. I will use the same tutorial videos used in phase one for the teacher training, for the learners' training. The learners' training will not interrupt normal lesson as it will be done after school hours. As many learners are computer literate, I will encourage the faster learners to teach the slower

ones. A further activity in this phase is for the participating teachers to pilot their lessons. These will be videotaped and reflected upon. The reflection will contribute to the final planning and design of the 4 final lessons that the three participants (my two colleagues and I) will teach.

Phase 3 - implementation

This phase consists of the implementation of the four planned lessons on the angle properties of geometric shapes for each teacher. These are: **Lesson 1:** angles formed within parallel lines, **Lesson 2:** angles in a triangle, **Lesson 3:** angles in a quadrilateral, and **Lesson 4**: angles in a complex shape (a combination of lesson 1, 2 and 3). A total of 12 lessons will thus be videotaped for analysis purposes.

Phase 4 – analysis of videos

In this phase the collected data will be collaboratively analysed by means of the stimulated recall method. According to Eskelinen (1991) one advantage of the stimulated recall method is that the method eliminates the problem of leaving out critical incidences which otherwise might have been forgotten. In addition, the analysis process using this method is flexible as one can stop and start the video recording at will. My two co-participant teachers and I will analyse each of our four lessons together and reflect on the role that GeoGebra played in each lesson using the analysis tool illustrated in table 1. The indicators that will be significant with regard to finding evidence for enhancing conceptual understanding are illustrated in table 2.

Phase 5 – teachers' perceptions and experiences

In this Phase I wish to conduct one-on-one interviews with my two co-participant teachers to follow up on what emerged in Phase 2. The focus will be to tease out the teachers' own perceptions and experiences about using GeoGebra as a visualisation tool to teach for conceptual understanding. The individual interviews will be semi-structured. According to Cohen, Manion & Morrison, (2007) a semi-structured interview is "where a schedule is prepared that is sufficiently open ended to enable the content to be recorded, digressions and expansions made, new avenues to be included and further probing to be undertaken" (p. 187).

Data collection

Observation

The distinctive feature of observation in a research process is that it offers a researcher the opportunity to gather live data from the naturally occurring social situations. "This direct cognition as a mode of research has the potential to yield more valid and authentic data than it would be with mediated or inferential methods" (Cohen et la., 2011, p.456). Observations of the 12 lessons (4 per participant) will be done by viewing the 12 video recordings collaboratively. To observe the role of GeoGebra visualisations, the observation will primarily focus on the indicators articulated in table 1. To observe the evidence of teaching for conceptual understanding the template in table 2 will be employed. By its very nature, observations have the risk of being selective and subjective, as stated by Nieuwenhuis (2011). To minimise this risk, the focus of the observation will be tightly framed by the two analytical instruments. In addition, all the three participants will be conscious of their own biases. It is envisaged that my co-participants and I will sit together at mutually convenient times and go through each video systematically using the two analytical instruments discussed below.

Interview

In Phase 4, I will use a face-to-face semi structured interview (Arksey & Knight, 1999) with each of the two teachers who participated in this study. The purpose of this interview is to reflect firstly on the intervention process and secondly on the observation/analysis process with special reference to the two research questions. The interview will be structured around a set of questions specifically related to the role of GeoGebra as a visualisation tool, and how it can be used as an effective teaching tool in developing conceptual understanding. The questions will be pre-determined yet open- ended. "Open-ended questions are flexible, they allow the interviewer to probe so that he/she may go into more depth or clear up misunderstanding. They allow the interviewer to make a truer assessment of what the respondent really believe" Cohen et la., (2011 P.416). The interview will also serve to probe deeper and seek clarifications where necessary. Holstein and Gubrium (2003) describe interviewing in qualitative studies as a unique form of conversation, which provides the researcher with empirical data about the social world – in this case the teaching experience of 4 lessons. All interviews will be recorded using a voice recorder and will be transcribed.

Data analysis

The analysis of my data will be multileveled. On one level my co-participants and I will collaboratively analyse the video recordings of the 4 lessons we each taught. The focus of this analysis will be to find evidence of the role that GeoGebra visualisations played in developing conceptual understanding. The template in Table 1 is the analytical instrument that will facilitate this process. The analytical instrument in Table 2 will facilitate the second level of analysis which focuses on how (if at all) the use of GeoGebra enhanced conceptual understanding. The template in Table 1 was adapted from Kosslyn's (1994), while Table 2 was adapted from Kilpatrick et al., (2001).

Table: 1 Analytic template A – Visualisation indicator

Adapted from Kosslyn (1994)

Visualisation processes	**The role of GeoGebra visualisations – external indicators** *Generate the image – inspect the image – transform the image – use the image*
Image generation	*Generate the image* This indicator refers to the initial image that the teacher generated in GeoGebra to develop the mathematical idea at hand. This image forms the basis from which the teacher will then manipulate certain elements and properties to either demonstrate or develop the mathematical idea further.
Image inspection	*Inspect the image* This indicator is about scanning, examining and scrutinising images in order to distinguish similarities and differences between them. I will therefore examine how the GeoGebra images are used by teachers to reinforce differences and similarities of various mathematical concepts. Similarities and differences can be identified on the display and discussed by the whole class. These differences and similarities can be demonstrated dynamically by manipulating the image. Mathematics discovery and concept understanding is thus enhanced.
Image transformation	*Transform the image* This indicator specifically refers to the transformation of an image. I will specifically look for how the teachers use GeoGebra to dynamically change and transform an image to demonstrate certain properties of angles in shapes. I will also be specifically looking for evidence of rotation, enlargement and translation of angles and shapes on the computer screen.
Image use	*Use the image* I will use this indicator to specifically look for evidence of how GeoGebra is used to emphasise and develop the appropriate properties of shapes and angles. For example, how does a teacher manipulate features of a rectangle to show that it is also a parallelogram?

Table: 2 Analytical template B-indicators of conceptual understanding

Adapted from (Kilpatrick et al., 2001)

Conceptual understanding indicators or themes	**Approaches to build conceptual understandings during teaching and learning. Description of indicators in relation to GeoGebra as a visualisation tool.**
Connecting ideas and concepts in mathematics	The teacher uses GeoGebra to demonstrate connections between multiple concepts and establish relationships. The teacher uses GeoGebra to explore the relationship between concepts and how they are linked. Teacher encourages learners to explore connections between selected concepts.
Connecting mathematics to real world	The teacher use GeoGebra to connect mathematics to real world examples. Every day-shapes are used and other properties are expressed.
Connecting mathematics to prior-knowledge	The teacher uses GeoGebra to construct dynamic-Geometry diagrams that are familiar to learners. The teacher makes use of what the learners already knows and draws from their past experiences.
Representing mathematical analysis in different ways	The teacher uses GeoGebra to represent mathematics in in different ways. The teacher uses diagrams and GeoGebra visualisation to illustrate geometric and algebraic properties. The teacher is able to drag around and change measurements, but maintaining the dependencies in construction. i.e generated different shapes will sustaining the same properties.

Justifying and explaining mathematical ideas and solutions	The teacher is using GeoGebra visuals to explore dependencies, relationships and proofs of the central concepts and theorems.

The interviews will be audio-recorded, transcribed and analysed to explore further the participant's experiences and perceptions when using GeoGebra as visualisation tool to enhance conceptual understanding. In the analysis of the transcripts, similar phrases and words will be put into categories and themes. These themes and categories will then be used to enrich my narrative that emerged from the analysis using the two Tables.

Validity

To enhance validity, the three participants will first each pilot a lesson. These lessons will be reflected upon using the analytical tools instruments above. Appropriate refinements will be made to the template in order to eliminate ambiguities. Validity in this study is enhanced by the collaborative design of the analytical process. "The outcomes of the [research] project are more accurate when participants are involved throughout" (De Vos, Strydom, Fouche & Delport, 2011, p. 8). The involvement of the participants in the analysis is also a form of member checking which enhances validity (Maxwell, 2009).

Ethics

See attached form.

Table: 3 Summary of data generation process and tools used

Tools	Purpose	Data generated	Analysis
Video tape for Observation purposes	To obtain in-depth information about **how teachers use GeoGebra as tool for visualization** to teach Geometric angle properties in Grade 9	Qualitative data Transcripts	Qualitative themes emerging from the interventions that describe the role GeoGebra visualisation
Interviews	To obtain further reflective data on **teachers' perceptions and experiences**.	Qualitative data Transcripts	Qualitative themes emerging from the teachers that address the research questions: how GeoGebra visualisations enhance conceptual understanding? What is the role of GeoGebra visualisations during the lesson presentation?

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RU FACULTY OF EDUCATION: ETHICAL APPROVAL APPLICATION

IMPORTANT: The following form needs to be completed by the researcher and submitted with their research proposal to the Education Higher Degrees Committee. The details to which this form relates should also be evident in the text of the proposal.

GENERAL PARTICULARS

MEd (Half thesis)		**MEd (Full thesis)**	√	**PhD**		**Other: Please specify**	

TITLE OF RESEARCH: An analysis of how GeoGebra can be used as a visualisation tool by selected teachers to develop conceptual understanding of the properties of geometric shapes in Grade 9 learners.

DEPARTMENT/INSTITUTE: Education/Rhodes University

DATE: [Submission to EHDC] December 2015

RESEARCHER: Erasmus Mwiikeni

SUERVISOR: Prof. Marc Schäfer

ETHICS

Respect and dignity

I will communicate the goal of my research to all the participants in the study. They will be informed that they have a right to withdraw from the research at any time. Pseudonyms will be used so that they remain anonymous. The school at which the research takes place will also remain anonymous.

Since my research will collect some data by means of video tape, participants will be assured of their anonymity in my thesis. In the event of wishing to use the videos in a conference or other professional presentation I shall request their written consent. Normal teaching time will not be disturbed because my research activities will be scheduled after school hours.

Transparency and honesty

Permission to conduct the research at my school will be obtained from the Director of Education in the Oshana Region, Namibia. Once the proposal is approved by Rhodes

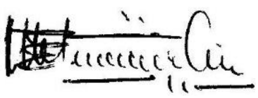
University and permission is granted from the Regional Director, I will then ask for permission from the principal and Head of Department of the school where the co-participant teachers are to be sourced. The two teachers will be informed about the purpose of the study and asked to volunteer to participate. They will then be asked to complete a consent form that they agree to be part of the research. Parents of learners taking part in the study will also be informed well in advance about the study. A written brief about the research will be sent to parents in their vernacular languages if necessary. In this document they will also be informed of their right to agree or disagree for their children to participate in the research. Data and other presentations generated during the research will be shared and discussed with participants.

Accountability and responsibility

I will be accountable for the entire research process and ensure that the data are kept safe. Although my position as a principal might be seen as a threat I will ensure that mutual trust amongst the participants and myself has been established before the research commences. This has already been established to a certain degree because, I know the participants and there is a good collegial relationship between us. There is a mutual passion and interest for technology amongst us. As the data analysis is done collaboratively – i.e. we will be observing each other's lessons, this will also build trust. Since the co-participants are mathematics teachers who are interested in technology, their participation will add value to their practice. During the research I am just a mathematics teacher like them.

Integrity and academic professionalism

Integrity will be upheld at all times in this research project. I will make sure that the research findings and all the data are presented authentically without any distortion or manipulation to suit my assumptions and opinions. All raw data will be kept safe and secure. I will declare that the entire final thesis is my own work and I will acknowledge and reference other people's work according to the Rhodes University guidelines for academic writing.



E. Mwiikeni- Researcher

Date: 17 November 2015

Place: Namibia- Ongwediva

Prof. Marc Schäfer -Supervisor

Date: 16 November 2015

Place: Grahamstown