RHODES UNIVERSITY
FACULTY OF EDUCATION
RESEARCH PROPOSAL

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Degree : Master of Education (Full thesis)

Department : Education

Field of Research : Mathematics Education

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Provisional Title

An analysis of the nature of visualization objects in three Namibian Grade 9 mathematics textbooks.

Abstract

Textbooks are a universal and central element of teaching and learning mathematics (Namibia. Ministry of Education [MoE], 2008). Steenpaβ & Steinbring (2014) state that diagrams in mathematics textbooks are often used as visualization objects to enhance learning of mathematical concepts. Visualization objects in textbooks are thus important teaching and learning tools (Fotakopoulou & Spiliotopoulou, 2008).

This Namibian interpretive case study is set out to analyze the nature of visualization objects used in the three approved grade 9 Namibian mathematics textbooks. I intend to use an analytic tool which specifically interrogates the type, the relation to content, the relation to reality, the function and dimension of each visualisation object found in the algebra and geometry chapters of each book. This study also includes a survey questionnaire and interviews with selected teachers on how they perceive and use the identified visualization objects. I also intend to interview one of the authors of the books to obtain his/her rationale for using the identified visualization in his/her book.

It is hoped that this study will provide some insights into the use and nature of the visualization objects in these textbooks, contribute to improving the quality of textbooks evaluations and design of suitable textbook evaluation criteria in Namibia, and create a critical awareness of the role of visualization objects in textbooks amongst teachers, policy makers and potential authors.

Common Statement:

This proposed research study is part of the “Visualisation in Namibia and Zambia” (VISNAMZA) project which seeks to research the effective use of visualisation processes in the mathematics classroom in Namibia and Zambia (Schäfer, 2015). Research in the VISNAMZA project is currently centred around 5 MEd studies and 1 PhD study.

Field of Research

Mathematics Education: Visualization

Provisional Title

An analysis of the nature of visualization objects in three Namibian Grade 9 mathematics textbooks.

Context

Textbooks are a universal and central element of teaching and learning mathematics, as it is for many school subjects (Nicol & Crespo, 2006; Namibia. Ministry of Education [MoE], 2008). In many developing countries where there is often a shortage of reading materials, the textbook is often the only resource text that teachers and learners can draw on for subject-specific content and information (Namibia. MoE, 2008) and thus their quality assumes special importance. Further, national and school examinations are often largely based on an ability to reproduce what is to be found in prescribed textbooks (*ibid*).

The character and quality of textbooks are important. They need to take into account the specific intended functions and also context in which they will be used (Nicol & Cresto, 2006). Therefore, the design of textbooks is critical for their users, i.e. to both teachers and learners. Gu, Huang & Marton (2004) state that in designing a quality mathematics textbook, the following vital elements should be considered: alignment with the curriculum, the content needs to be mathematically correct, coherently developed and should have good topical examples (p.13). The approach that the textbook adopts and promotes needs to be stimulating and creative. Textbooks need to draw on mathematical history, and be driven by research on teaching and learning mathematics. The textbook needs to be age-appropriate, have an appropriate number of exercises and aligned with the relevant assessment policy. The quality of the language, visuals objects such as illustrations and graphics are also critical elements that need to be considered *(ibid).*

Researchers indicate that diagrams in mathematics texts, such as textbooks, are often used as visual objects to enhance the learning of mathematical concepts (Konyalioglu 2003; Guzman 2002; Steenpaβ & Steinbring, 2014). These visual objects can illustrate an abstract idea in a concrete way or they can reinforce a mathematical procedure diagrammatically. According to Steinberg (2005) and Sobbeke (2005) taking a visual approach to teaching and learning mathematics can be meaningful and effective. Although visual objects in textbooks are

important teaching and learning tools, only recently have they attracted research attention (Fotakopoulou & Spiliotopoulou, 2008). According to Zimmermann & Cunningham (1991, p. 3), "mathematical visualization is the process of forming images and using such images effectively for mathematical discovery and understanding". Similarly, in his discussion on the roles of visualization, Arcavi (2003) defines visualization as:

> The ability, the process and the **product** of creation, interpretation, use of and reflection upon **pictures, images, diagrams**, in our minds, **on paper** or with technological tools, with the purpose of depicting and communicating information, thinking about and **developing previously unknown ideas and advancing understandings**.

For the purpose of this study, I choose to focus on the efficacy of visualization as a product. These products will be referred to as visual objects and include diagrams, sketches, pictures, graphs and illustrations. I wish to research visualization processes that are embedded in visual objects in mathematics textbooks that promote the development of mathematical proficiency and understanding (Kilpatrick, Swafford & Findell, 2001). This study will be exploring these objects in relation to conceptual understanding and procedural fluency (Kilpatrick et al., 2001). This study will be analysing the nature of visualization objects in the three approved Grade 9 Namibian mathematics textbooks.

Before examining the literature on visualization and visualization objects, a brief outline of the Namibian selection process of mathematics textbooks is provided.

As a procedure in Namibia "all textbooks in education require the evaluation and approval of the Namibia National Institute for Educational Development (NIED)'s curriculum panels" (Namibia. MoE, 2008, p. 4). The Namibian curriculum and learning support materials review policy and review cycle of 2015 lists reasons for selecting textbooks for content subjects such as mathematics (Namibia. Ministry of Education, Arts and Culture [MoEAC]. 2015). Textbook evaluators mostly base their selection on four criteria: the conformity to the subject syllabus; the appropriate coverage of the prescribed content; the language and editorial quality and lastly on the design, presentation and ease of use of the textbook (Namibia. MoEAC, 2015). Although the use of visualisation is not specifically mentioned, it is partly incorporated in the last criterion, which looks at the usefulness, relevance and accuracy of graphics and illustrations, quality and

attractiveness of illustrations. This criterion, in my view, does however not sufficiently and explicitly articulate the desired characteristics of visualization objects that are required. I argue that the criterion is not substantial enough to assess the appropriateness and quality of visualization objects in mathematics textbooks. This thus suggests that there is a gap in the Namibian evaluation criteria for assessing visualization objects in mathematics. I therefore envisage that this study may *inter alia* contribute to improving the quality of textbooks evaluations and design of suitable evaluation criteria in Namibia.

The textbook review policy states that, only three titles per grade system are approved and make it to the textbook catalogue (Namibia. MoEAC, 2015). In 2008 NIED's (Namibia. MoE, 2008) curriculum panel approved three mathematics learner's textbooks for grade 9. Two are written by Namibian authors and the third by a South African author. These are the books that are central to my research project. They are called: 'y = mx+ c To Success', 'Mathematics for Life Grade 9' and 'Discover Mathematics Grade 9'. All these three mathematics textbooks cover the seven topics that are stipulated in the Grade 9 Namibian mathematics syllabus. Some of them have expanded the topics by including subtopics in their chapters. Table 1. Illustrates the topics presented in the three textbooks and the number of visualization objects used in the chapters to be studied, the shaded topics are under study.

TABLE 1: Topics presented in the three Namibian Mathematics grade 9 textbooks and the number of visualization objects (VO's) evident in the chapters to be studied

Topics stipulated in the grade 9 Syllabus	**y = mx+ c**	**Number of VO's**	**Maths for Life 9**	**Number of VO's**	**Discover Maths 9**	**Number Of VO's**	**Total**
Numbers and Operations	✓		✓		✓		
Money and Finance	✓		✓		✓		
Algebra	✓	15	✓	9	✓	17	41
Geometry	✓	89	✓	76	✓	60	225
Graphs	✓		✓		✓		
Mensuration	✓		✓		✓		
Statistics and Probability	✓		✓		✓		

In 2007, a survey of 13 schools in Namibia for grade 5-12 revealed that textbooks to learner ratios for the core subjects of English, mathematics and science averaged nearly 1 textbook to 5 students (Namibia. Millennium Challenge Account [MCA], 2009). A shortage of books and other instructional materials was prevalent in Namibian primary and secondary schools in those years

(Namibia. MoE, 2008). In July 2008 the government of the Republic of Namibia and that of the United States of America (USA) reached an agreement whereby the USA government provided grant funding for public investment in Education through the Millennium Challenge Account (MCA) for five years. In the education sector the MCA aimed to improve the quality of general education and access to and management of textbooks (Namibia. MoE & MCA Namibia, 2010).

The objective of the Textbooks Activity in the Education Project of the MCA, Namibia is "to give all learners access to a textbook for each priority subject" (Namibia. MoE, 2009, p. 3). "this project focuses on the procurement system and procedures as well as textbook management and utilization at school level" (Namibia. MOE, 2010, p.3). The overall objective of this sub-activity in education was to achieve a textbook to learner ratio of 1:1 by 2013 for core/key subjects. These were in the first instance: English, mathematics and science at primary and junior and secondary levels (Namibia. MOE, 2008). At the end of this project the report (Namibia. Millennium Challenge Account [MCA], 2015) indicated a great improvement in access to textbooks, and the achievement of the goal of 1:1 learner to textbook ratio in the three core subjects was reached.

Indeed this is evident in the school I work in as well as in other neighbouring schools that form the Oupumako cluster. The Grade 9 learners of the rural school I work in and the other schools in this cluster have one or more of the three recommended mathematics textbooks. These textbooks are typically the only books that the learners have access to (Namibia. MoE, 2008). The textbooks are used in the mathematics classrooms during the lessons, examination preparations, and at home for homework and study purposes.

Conceptual framework

Keeping in mind the visualization definition by Arcavi on page 3 of this proposal, my analysis of textbooks is informed by four conceptual constructs: visualization as a product, visualization objects, conceptual understanding and procedural fluency.

Many researchers record that the use of visualization objects are central elements to the effective teaching and learning of mathematics (Vincent & Stacey, 2009; Steinbring & Gellet 2014; Dimmel & Herbest, 2015). Arcavi (2003) suggests that visual representations help us to 'see the unseen'. Likewise Sobbëke (2005) proposes that in mathematics classrooms, visual

diagrams help the learners to better see mathematical concepts and ideas. Steinbring & Gellert (2014) affirm that visualization representations can be used as an epistemological tool to explore mathematics structures and bring about new meaning. Visual objects are “especially important for textbooks which aim to lead non-expert readers to an understanding of the esoteric domain of a scientific field” (Fotakopoulou & Spiliotopoulou, 2008, p. 321).

Duval (1999) suggests that “invisible mathematical tools need other forms of representation to become communicable” (p.3). They can be presented as visualization objects, which include drawings, pictorial illustrations/pictures, diagrams, charts and graphs (Fotakopoulou & Spiliotopoulou, 2008). Kim (2012) observes that “more pictures, illustrations, and diagrams have been used in recent textbooks than in the past” (p.175). Kim’s (2012) study finds that visual representations in mathematics textbooks can serve not only as informative agents but also as ‘‘tools for thinking’’ which students manipulate (p.178). Many researchers deem the use of visualization objects to be a powerful tool in learning and understanding mathematical concepts (Fotakopoulou & Spiliotopoulou, 2008; Kim, 2012; Steinbring, 2014). Kilpatrick, Swafford & Findell (2001), assert that learning with understanding is more powerful than simply memorizing because it improves retention, promotes fluency, and facilitates learning related material (p.113).

Kilpatrick et al., (2001) define “conceptual understanding as a comprehension of mathematical concepts, operations, and relations” (p.116**).** They further define procedural fluency as “knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently” (p. 121). Moreover they claim that conceptual understanding and procedural fluency are interwoven. For instance, a certain level of skill is required to learn mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding *(ibid).* The use of visualisation objects is integral to the process of understanding mathematics. As Levin & Mayer, as cited in Kim, (2012) suggest that “since visual representations, provides students with concrete and concise images of related concepts, they help improve students’ understanding of the contents” (p. 177).

In contrast Steinbring & Steenpaβ (2014) however warn that mathematical visual images can be ambiguous elements that do not necessarily convey the concept effectively to students, and can thus lead to misunderstanding. Steinbring & Gibert (2014) add that “interpreting mathematical visual diagrams is a challenge faced by mathematicians as well as students” (p.16). Therefore,

students require appropriate assistance and guidance from teachers and knowledgeable peers as they select, interpret, and create visual models of mathematics (Moyer & Jones, as cited in Moyer, 2014, p.3). For these reasons, it is important to understand how representations are used in mathematics textbooks because ‘‘representation is more than a process; it is a way of teaching and learning mathematics’’ (Fennell & Rowan, as quoted by Kim, 2012, p.1).

To better understand visualization objects in grade 9 mathematics textbooks, the adopted and modified framework illustrated in figure 1 will be used.

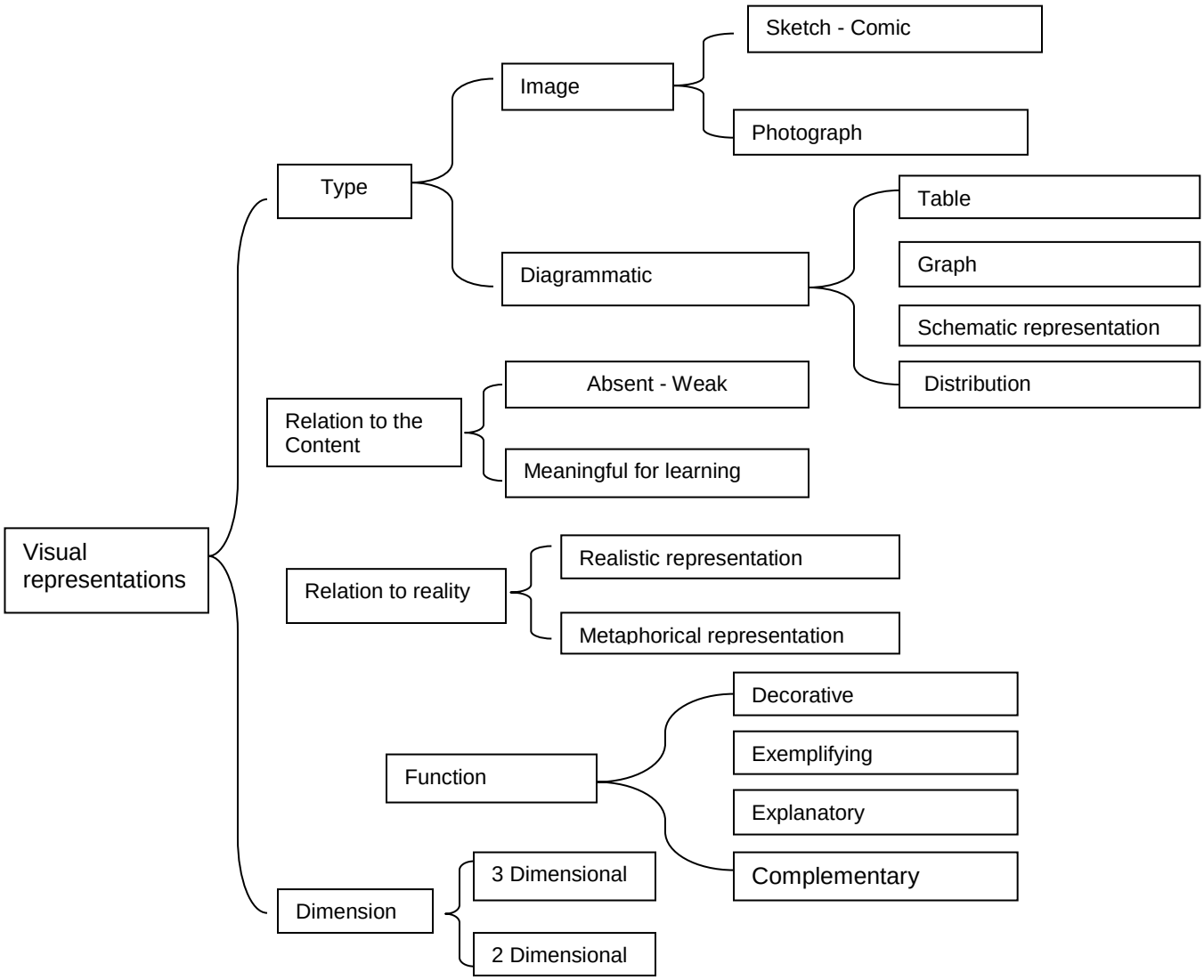


Figure 1: Analytic tool to analyse visualization objects
(Source: Adapted from Fotakopoulou & Spiliotopoulou, 2008.)

Visualization objects can be categorized into many groups. For the purpose of this study they will be categorized according to their type, relation to the content, relation to reality, function and dimension (Fotakopoulou & Spiliotopolou, 2008), see figure 1. In terms of their type, the objects can be categorised into two groups: as an image or as diagrammatic. An image refers to any sketch, picture, comic or a photograph while diagrammatic visuals are like tables, graphs or schematic representations. In terms of relation to the content, the visualization objects can either be meaningful or irrelevant to the content.

In terms of their relationship to reality, visualization objects used can be classified as realistic, or as metaphoric (Fotakopoulou & Spiliotopoulou, 2008). That is to say, visual diagrams can either represent something in the real world or something that has no real existence, but only illustrate reality. In terms of the function of visualisation objects they can be used for decoration i.e. to make the page look attractive and interesting. They can be used to exemplify an idea or explain a concept. They can also be used to complement a definition or explanation. Finally, a visual object can be represented either in 2-dimensions or 3-dimensions.

Theoretical considerations

From a constructivist point of view, “all knowledge is constructed” (Nodding, 1990, p.7). Constructivism proposes that “we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge” (Simon, 1995, p. 115). Simon further argues that constructivism as an epistemological theory, asserts that knowledge can be developed whether or not there is a teacher present or teaching is going on (p. 116). Therefore, students need to make their own sense of the mathematical visual objects in textbooks to help them explore mathematical structures and generate new meaning (Steenpaß & Steinbring, 2014). In agreement with this Yackel (2001), a radical constructivist, alludes that “students construct their own meaning from the words or visual images they see or hear” (p.41). Hence, students need various resources, such as books, technological devices and media with visualization objects to construct knowledge. Textbooks are one of these and are traditionally a key resource to teaching and learning mathematics worldwide. In most cases the textbooks are the most readily available and accessible resource for teachers and learners in the classroom. Thus textbooks are a necessary and critical means for knowledge construction.

To understand how learners learn, Bruner (1990) developed three stages of representation: the enactive mode, the iconic mode and the symbolic mode. “These three modes of representing our experience are considered important to the development of children's understanding” (Drew, 2007, p. 20). The enactive mode involves representation of ideas through taking some form of action such as manipulating physical objects, and perform some physical task. The iconic mode, which is significant for this study, involves representing those ideas using pictures or images (Drew, 2007). The symbolic mode involves ideas represented through language or symbols usage, whereby learners are expected to judge and think critically (Bruner, 1990). Drew (2007) propose that the use of physical resources, models and images in mathematics teaching and learning relate well to the enactive and iconic modes of representation. The iconic mode thus resonates well with the purpose of my study.

Research Goals & the significance of the study

The aims of this study are twofold. Firstly to analyze the visualization objects used in three grade 9 Namibian mathematics textbooks in terms of their type, relation to content, relation to reality function and dimension. Secondly to understand selected mathematics teacher’s views about visualization objects as well as to describe the choices that selected authors made in selecting those objects in their textbooks.

It is hoped that this study will create a critical awareness of the role of visualization objects in textbooks amongst teachers, policy makers and potential authors. It is further hoped that the results of this study will inform the evaluation process of textbooks in Namibia and ensure a more comprehensive assessment of these resources.

Research questions

Having the aforesaid goals in mind, my research questions are:

1. What is the nature of different visualization objects evident in the Namibian grade 9 mathematics textbooks?
2. How are these visualization objects viewed in terms of their use by selected teachers in their teaching?
3. What were the author’s rationales for using the identified visualization objects in their textbooks?

Methodology

Orientation

This case study research is underpinned by an interpretive paradigm using a mixed method approach, viz. qualitative and quantitative methods (Bertram & Christiansen, 2014). Interpretive paradigm researchers "do not predict what people will do, but rather describe and understand how people make sense of their world and how they make meaning of their particular actions" (Bertram & Christiansen 2014, p. 26). A case study is well suited to "make sense of feeling, experiences, social situations, or phenomena as they occur in the real world, and therefore study them in their natural settings" (Rule & John, 2011, p. 60).

The choice of this paradigm is aligned with the purpose of this study which is to understand how teachers make sense of visualization objects and how textbook authors made their choices of these objects in their mathematics textbooks.

Research Method

This study takes the form of a case study, which I regard appropriate for my research project because I am looking at a specific, real life phenomenon in a specific context (Rule & John, 2011). The case in this study consist of three grade 9 Namibian mathematics textbooks that are approved by the NIED Curriculum panellists, 54 grade 9 teachers of Oshana region and two authors. The unit of analysis is threefold, namely the nature and roles of visualization objects in three mathematics textbooks and the perceptions of teachers and authors.

A case study is defined as a "systematic inquiry into an event or a set of related events which aims to describe and explain the phenomenon of interest" (Bromley, 1991. p.302). In addition (Schumacher & McMillan, 1989 as quoted in Maree, 2011) state that a case study does not necessarily mean that it is only one site that is studied, they can be more. A case study is said to be valuable, because it allows researchers to develop and present an in-depth view of a particular situation, event or entity (Rule & John, 2001, p. 77). At the same time a researcher needs to keep in mind some weaknesses associated with the use of a case study. Nisbet & Watt (1984, as cited in Cohen et al., (2007) state that in a case study "the results may not be generalizable except where other readers see their application, they are not easily open to cross

checking, they may be selective and are prone to problems of observer bias, despite attempts made to address reflexivity"(p. 256). I will address these under the validity section.

Research design

It is planned that this research will unfold in four phases.

During **Phase one**, I will refine the analytical tool and pilot it with a few visualization objects from the Maths for Life textbook. This will enable me to see if the analytical tool is reliable and effective to generate appropriate data to answer research question 1. At the same time I will develop and send out a survey questionnaire to all the 54 grade 9 mathematics teachers in the Oshana Region.

In Phase two, the analysis of the identified three textbooks using the refined analytical instrument will be done. Also I intend to collect and collate the completed survey questionnaire. I will then analyse the survey questionnaires using descriptive statistics. This will give me an overall picture of teacher's views and understanding of the importance and functionality of visualization objects used in the three textbooks. This will help to answer research questions 1 and 2.

Phase three I will be conducting telephonic or skype interviews with at least one of the authors of the three identified textbooks. This will help to answer research question 2 and 3.

Finally, in **Phase four** the analysis of all the transcribed interviews using qualitative analysis processes will be done.

Selection of Participants

The selection of the textbooks for this study was done using a 'purposive sampling strategy' (Bertram & Christiansen, 2014, p.60). In purposive sampling the researcher makes specific choices about the objects or people to include in the sample. I choose to use all the three approved mathematics textbooks for the junior secondary phase. Two chapters from these books will be analyzed, namely; algebra and geometry. From my teaching experience as well the Mathematics congresses I have attended, these two chapters are perceived to be

particularly challenging, poorly performed and not well understood by both teachers and learners.

For the survey questionnaire I have selected all the 54 grade 9 mathematics teachers in the Oshana region – the region in which I teach. Thus this was also done purposively. Furthermore the other participant will be two of the three authors of the selected textbooks, depending on the one I will be able to track down.

Techniques/ Tools

For the purpose of this research study, a number of data collection tools such as textbooks, my analytic tool, survey questionnaires and interviews will be used. Bertram & Christiansen (2014) suggest that a researcher needs to use a "fitness for purpose approach" whereby data collection methods need to match the kind of data he/she wants to collect (p. 41)**.** In this case a collection of all visual objects from the three grade 9 mathematics textbooks, y = mx + c, Discover Mathematics and Mathematics for Life in the specified chapters; algebra and geometry will be done by copying the visual objects and pasting them into organised documents.

I deem survey questionnaires as an appropriate tool for my purposes as this will enable me to collect data from 54 grade 9 mathematics teachers in Oshana region. In the questionnaire I will ask questions that seek teachers' general views and understanding of the visualization objects presented in the respective textbooks. I will also select a sample of pictures from these textbooks and set questions based on these objects to ascertain how they are used by the teachers answering the questionnaire. I will also seek the teachers' views on the appropriateness of the visualisation objects and what they would use instead of these objects.

Bertram & Christiansen (2014) emphasize that an interview is categorized to be a useful data collection method as it allows the researcher to ask more probing and clarifying questions. I intend to interview the authors of the books to obtain their views and explanations of the choices of visual representations that they used (or included) in their respective textbooks.

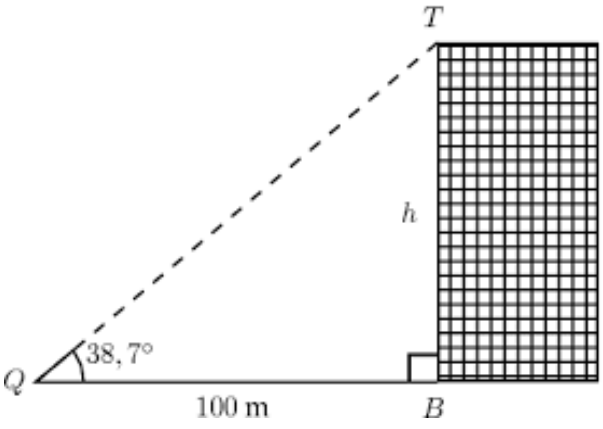
Data Analysis

Firstly I will analyze all 266 visualization objects in two chapters of the selected mathematics textbooks using my analytical framework illustrated in Figure 1 above. The framework has been transformed into a coding template as illustrated in Table 4 below. The coding scheme that I will use for each category of my template is illustrated in Table 3 below.

Table 3: Coding scheme for analysis of the visual objects.

Score	**Description of evidence of the indicator**
n/a	Not applicable
0	Weak evidence
1	Average evidence
2	Strong evidence

To illustrate the use of my analytical tool and coding, I provide a hypothetical example.

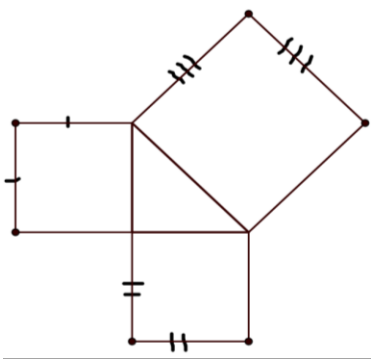


Problem: Use the trigonometric ratios to find *h*

Table 4: Coding template to analyse each visualization object

Category	Subtype	Definition	Score A
Type	Picture, Sketch/ drawing	A simple drawing/shape which does not have many details	2
	Distribution	Histogram; Point Graph; Stem-and-Leaf Plot; Tally Graph	n/a
	Photograph	A picture produced using a camera	n/a
	Table, matrix, text chart	A table of different sizes and forms	n/a
	Graph	A graphical representation of data, relations and processes in bar, column, pie, spider graphs, graphic timetable form.	n/a
	Schematic	A representation of an idea or theory for easy understanding.	2
Function	Decorative	Made to look more attractive or ornamental – a decoration	1
	Exemplifying***	These are diagrams that exemplify the written text. They make the text and the mathematical idea clearer.	2
	Explanatory	These are notes accompanying the diagram, which assist the text and provide new aspects of information necessary to make the idea under consideration clearer. The diagram is used to explain the main mathematical idea.	2
	Complementary	These are diagrams that provide information not included in the text nor described explicitly in the written form. The diagram complements the test and adds to or reinforces the written text	2
Reality	Realistic	Realistically represents the concept true to real life situation	2
	Metaphorical	Not having real existence	0
Properties	3D	It is three dimensional	0
	2D	It is two dimensional	2
Content	Absent/weak	It has a weak connection to the content	0
	Meaningful to the content	In line with the content	2

The subtype ‘Exemplifying’ needs further explanation. For example, a typical textbook, including the ones under scrutiny uses the diagram below to illustrate Pythagoras’ theorem which states that “In a right-angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides”.



My evaluation under the subtype ‘exemplifying’ might be that this visualization object only partially illustrates, or makes clearer the idea given in the text above. It does not fully exemplify the text, ie Pythagoras’ theorem, because it does not fully show in a visual sense what the text has stated as it does not illustrate that the square on the hypotenuse is the sum of the squares on the other two sides. If, for example, the diagram had included 5:4:3 ratios on the sides then it would exemplify the text statement more fully.

The scores in the fourth column will then be counted and illustrated using descriptive statistics.

The responses to the survey questionnaires will be analyzed using descriptive statistics. Pietersen & Maree (2011) define descriptive statistics as “a collective name for a number of statistical methods that are used to organize and summaries data in a meaningful way” (p.183). They further state that descriptive statistics enhances the understanding of the properties of the data and data can be represented in a graphical or numerical way (*ibid*).

An inductive approach will be used to analyze the data from the transcribed interviews of the authors. (Maxwell, (2005), as cited in Fotakopoulou & Spiliotopoulou, 2008) states that there are three main categorizing strategies of qualitative analysis where the categories, themes and pattern emerge from data, which fits with my study. These categorizing strategies are namely: organizing data into categories identify patterns and generate themes and concepts. I will organize the data collected into categories and identify patterns among the categories. These categories will make it easier for me to develop a general understanding of what is going on, to generate themes and concepts. This will enable me to obtain a deep insight on the views and opinions of the authors on the visualization objects under scrutiny.

Summary Table of the Research Process

Phases	Instrument	Purpose	Data	Analysis
Phase 1	Consent letters	To seek permission from the Director of Education Oshana region.	Not applicable	Not applicable
	Analytic tool	The development of the analytic tool to analyze visualization objects	Not applicable	Not applicable
	Select a few visualization objects from one text book and pilot the analytical tool	To analyze them and to ensure validity	The visual objects: nature, function, properties, relation to reality, relation to	Analytic tool that was developed

			content.	
	Survey questionnaires	Development of the survey questionnaire for the Oshana region teachers and send them to teachers.	Not applicable	Not applicable
Phase 2	Analytic tool	Analyze and categorize the visualization objects of the selected textbooks	The nature, type, function, the relation to reality and relation to the content of those objects.	Analysis of visualization objects using analytical tool that was developed
	Survey questionnaires	Collect and gather completed survey questionnaires	The views of teachers on the visualization objects used in the three textbooks	Quantitative descriptive statistics
Phase 3	- Interview	Design the interview questions and conduct it with two authors.	Authors rationale on the selection of the visualization objects used.	Qualitative analysis using inductive approach

Validity

To ensure validity of this study, I will firstly pilot the analytical tool on one textbook 'Maths for life' to help me find out if this analytic tool is appropriate and effective in terms of my research questions. In addition, I will also have my questionnaire reviewed by my supervisor before piloting them. I will then pilot the questionnaire with two mathematics teachers at our school before sending it out to the 54 grade 9 mathematics teachers. Furthermore, to enhance credibility during the interviews I will make use of an audio-recording device to gather a full record of the interviews. I will explicitly explain the aims and objectives of this study to all participants to enhance their understanding of this project and thus ensure alignment of their responses to the goals of the study. Maxwell explains that "triangulation reduces the risk of chances associations and systematic bias, and relies on information collected from a diverse range of individuals, teams and settings, using a variety of methods" (1996. p.93). In this study I will make use of triangulation by collecting data using a variety of methods and getting information from many different participants as discussed earlier.

Ethics

See attached form

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RU FACULTY OF EDUCATION: ETHICAL APPROVAL APPLICATION

Med (Half thesis)		**Med Full thesis)**	×	**PhD**		**Others: Please specify**	

Title of research: An analysis of the nature of visualization objects in three Namibian Grade 9 mathematics textbooks.

Department/Institute : Education Department
Date : 15 October 2015
Researcher : Selma Ndilipomwene Nghifimule
Supervisor : Professor Marc Schäfer

ETHICS

Respect and Dignity

The participants will be well informed about their voluntarily participation in the study as well as their freedom to withdraw at any time. Participants' identity will remain anonymous. As the textbooks are in the public domain it will however be impossible to keep the name of the authors anonymous. They will be made aware of this which might influence their decision to be interviewed or not. I will ensure that there will be mutual respect. All the data will remain confidential between me and my supervisor. Consent to use a voice recorder during the interviews will be obtained from all the participants'. All the planned research activities will take place after formal classes.

Transparency and honesty

Formal consent will be sought from the director of education in the Oshana region, authors, teachers and their principals. The nature, purpose and what this study entails will be clearly explained to all participants. In terms of the authors I will specifically clarify that the research is not about the comparison of textbooks rather it is an analysis of the nature of the diagrams and about how teachers use and perceive these in the respective textbooks.

Accountability and Responsibility

All the data will be treated with sensitivity and will be kept securely. I am aware of my position as a mathematics facilitator in the region, and I will assure the participants that my position will not compromise any of their responses.

Integrity and Academic professionalism

This research will be my own work, using my own words and where I draw on others works or ideas, I will appropriately acknowledge and reference them. In addition, collected data will not be manipulated, compromised, fabricated or misreported, rather they will be presented as they are.

_Selma _Nghifimule _Marc Schafer_
Signature (researcher) Signature (supervisor)
Date 14 October 2015 Place**:** NIED, Okahandja