RHODES UNIVERSITY, FACULTY OF EDUCATION
RESEARCH PROPOSAL

An analysis of visualization processes used by selected Grade 11 and 12 learners when solving algebraic problems.

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Degree : **Master of Education (Full thesis)**

Department : **Education**

Field of Research : **Mathematics Education**

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Abstract

Visualisation is gaining visibility in mathematics education research. It is a powerful tool for solving different types of problems in many areas of mathematics, including Algebra – the mathematical domain of this study. This case study aims to analyse the visualisation processes that students use to solve selected algebraic problems. Six grade 11 and 12 students will be purposely selected to participate in this study based on their mathematics capabilities and also willingness to participate in the study. The participants will be video recorded as they work through 10 items of an Algebraic Visualisation Tasks (AVT) worksheet. They will also be interviewed about the visualisation processes they employed when solving each of the tasks. The videos and the interviews will be analysed with the aid of an adapted visualisation template.

Common statement

This proposed research study is part of the “Visualisation in Namibia and Zambia” (VISNAMZA) project which seeks to research the effective use of visualisation processes in the mathematics classroom in Namibia and Zambia (Schäfer, 2014) Research in the VISNAMZA project is currently centred around 5 MEd studies and 1 PhD study

October 2015

FIELD OF STUDY: Visualisation in algebra

PROVISIONAL TITLE: An analysis of visualization processes used by selected Grade 11 and 12 learners when solving algebraic problems.

CONTEXT

Makina and Dirk (2009, p. 58) argue that "Mathematics, as a human and cultural creation dealing with objects and entities quite different from physical phenomena, relies heavily on visualisation in its different forms and at different levels". Watson (2007) asserts that the use of diagrams, visual representations and mental images are important elements in solving mathematical problems. In as much as the ability to solve problems is at the heart of mathematics, visualization is at the heart of many mathematical problems solving (Ho, 2010). There are many ways to solve algebraic problems and learners need the skill to choose an appropriate visualization strategy for a specific problem (Polya 1959; Ho, 2010).

Presmeg (1995) articulates that visualisation includes processes of constructing and transforming both visual and mental imagery. Zimmermann and Cunningham (1991) define visualisation "as the process of producing or using geometrical or graphical representations of mathematical concepts, principles or problems, whether hand drawn or computer generated."(p.1). According to Chiappini & Bottino (2010) we use visualisation to refer to the complex phenomenon of visual imagery that plays an important role in all meaning-making and understanding, as well as in all reasoning (p.2).

Arcavi (2003) defines visualisation as:

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p.217).

Visualisation is gaining increased visibility in mathematics education (Arcavi, 2003).Its significance and the role that it plays in mathematics teaching and learning is noted by many researchers (Arcavi, 2003; Makina & Wassels 2009 and Bhagat & Chang, 2015). Makina (2010) affirms that visualisation is very important in "teaching for understanding" in

Mathematics because it helps teachers to engage learners in realistic situations and with facilitation of lessons. Arcavi (2003, p. 215) articulates that visualisation offers “a method of seeing the unseen and we are encouraged and should aspire to ‘see’ not only what comes ‘within sight’, but also what we are unable to see”.

The literature suggests that it does not appear to be the mathematical knowledge or computation skills alone that are key to mathematical understanding but other contributing factors including the ability to visualise the problems which may affect achievement (Cummins et al., 1988; Hegarty et al., 1995; Kyttala & Bjorn, 2014).

In my experience many students often struggle to use visualisation as a strategy especially when solving mathematical problems. This, I argue, could negatively affect their performance. Healy and Hoyles (1996) articulate that “unlike mathematicians, students of mathematics rarely exploit the considerable potential of visual approaches to support meaningful learning. …they are reluctant to engage with visual modes of reasoning” (p. 67). Healy and Hoyles continue by stressing the advantages of being able to use particular images or diagrams in the service of mathematical generalization, and of making connections between modes of thinking. Van Garderen, Potch & Scheuermann, (2013) noted the importance of employing visualisation strategies. They argue that the use of visual aids such as sketches and diagrams are very powerful strategies in solving different types of problems for many topic areas.

Thornton (2000) proposes that “though much has been said about visualisation in general, there are still many issues concerning visualisation in mathematics education, which require careful attention.” (p. 251)**.** There is a need for teachers to understand how learners think visually during problem solving to be able to teach mathematical concepts such as algebraic concepts and problem solving (Makina & Wassels, 2009). It is important to listen to learners since there is a difference between what we (as teachers) want children to learn and what they actually take from our lessons (Makina & Wassels, 2009). Makina and Wassels further state that understanding the student’s mind during problem solving improves the teaching of mathematics. Steenpaß, et al., (2014) resonate with Makina & Wassels that “understanding how learners come to understand and conceptualize the world around them can help teachers facilitate learning” (p. 2).

Therefore my proposed research project will not only analyse the visualisation strategies employed by learners in solving algebraic problems but it will also analyse how the learners use these strategies and identify the visualisation processes that they employ. Although the literature seems to use the terms for visualisation processes and visualisation interchangeably, for the purpose of the study I will commit to the former.

Visualisation processes stimulate learners to ‘‘discover the unexpected, and describe and explain the expected’’ (Thomas & Cook, as cited in Rivera et al., 2014, p. 1). A visualisation process is one which involves visual imagery with or without a diagram, as an essential part of the method of solution (Presmeg, 1985, p. 298). The interaction of context, visual representational forms, and using technological tools is seen as key strategies that support functional understanding (Confrey & Smith, 1994). Using visualisation processes can thus assist learners to make new discoveries and solve mathematical problems. It is therefore important for teachers and learners to be aware of the type of visualisation processes and strategies used by learners that help them in solving algebraic problems. Ho (2010) articulates that “if visualization is at the heart of mathematical problem solving, then it is vital that both teachers and students see the role of visualization clearly and use it to help them in their problem-solving process”(p. 3).

Threlfall (2009) defines problem solving strategies in a mathematical context as the “different ways” that mathematical problems are solved (p. 154).When learners are given a mathematical problem the overall approach to the problem is what Threlfal (2009) and Ashcraft (1990, as cited in Threlfall, 2009) refer to as strategies. For example, we can examine the possible responses of learners to a problem such as “*a number is trebled and then 7 is added to it. If the total is 28, find the number*”. When learners are asked how they solved the problem, their responses will be manifold. They may say:, “*I used the method that I was taught by my teacher*” or “*I came up with an equation that I solved to get the answer”* or “ *I did the calculation mentally*” or “*I used the try and error method*” or “ *I use a sketch to visualise the problem*”. All these are legitimate strategies that learners use to solve the problem. The conclusion being that there are many different approaches that lead to the same solution.

Good and appropriate strategies and representations enable learners to solve problems efficiently and accurately (Heinze, Star & Verschaffel, 2009). Rivera (2003, p.59) postulates that “visual strategies play a mediating role in the emergence of children’s sophisticated, structured and necessary understandings of mathematical objects”. Children use visual strategies to help them conduct explorations, organize relevant data and anticipate an intended analysis (*ibid*).

Draper (2010) argues that “visualisation strategies help learners recall facts, get the main idea, make an inference, draw a conclusion, predict/extend and evaluate” (p.11). Leaners use visualisation strategies to “analyse and make conjectures about information, to analyse situations to make connections and plan solutions” (Draper, 2012, p. 2). However, individual learners should acquire the ability to solve mathematical tasks flexibly by a diversity of meaningful acquired strategies and representation (Heinze et al., 2009). Learners with strategic competence can not only come up with several approaches to a non-routine problem but can also choose flexibly the method to suit the demands presented by the problem and the situation in which it was posed (Kilpatrick.et al., 2001, p. 129).

Learners can only use visualisation processes flexibly and adaptively when they know when to use them, why to use them and where to use them. In short, the choice of strategy is an important consideration. My proposed research therefore aims to explore the visual processes that selected learners use to solve algebraic problems. I believe that this will benefit other teachers and learners in tackling similar problems and come up with new strategies to solve mathematics problem in future.

Past and current research focusing on visualisation strategies was reviewed in order to inform the proposed research .Of particular interest to my proposed research is the research findings by Ho (2010), who researched on how visualization helps students in mathematical problem solving. Ho (2010) documented five processes that learners may go through when solving mathematics problems. These are: **understanding** the spatial relations of the elements in the problem**; connecting** to a previously solved problem**; constructing** a visual representation (in the mind, on paper, or through the use of technological tools)**; using** the visual representation to solve the problem and **encoding** the answer to the problem. These processes are important and play a vital role in the visual representation process as they contribute to the organisation of the thinking and problem solving processes. These processes “help learners to acquire …a certain intuition of the abstract, a set of mental reflexes, a special familiarity with the object at hand that affords them something like a holistic, unitary and relaxed vision of the relationships between the different objects of their contemplation (Guzm´an, 2002, p. 1).

Furthermore, Polya (1957) identified four basic principles of solving problems. The first principle is *to understand the problem,* the second is *to devise a plan*, the third principle: is to *carry out the plan* and the fourth: *to look back.* Polya (1957) adds that though there are many ways to solve algebraic problems, the skill of choosing an appropriate strategy is best learned by solving many problems. That means learners should practice and solve a multitude of mathematical problems in order to gain experience in choosing appropriate problem solving processes.

Theoretical considerations

I position my proposed research within constructivism and the theoretical work of Cobb (1989) and Bruner (1966). According to them a constructivist view of learning mathematics advocates that students construct their own mathematical knowledge rather than receiving it in complete form from the teacher or a textbook (Perry, Goeghegan, Howe, & Owens, 1995). The knowledge construction or reconstruction is largely independent of the way students are taught. Students construct their own knowledge for themselves resulting in their own understanding (Ndlovu, 2013). Von Glasersfeld (2001) argues that from a constructivist point of view "human beings can only know what they themselves have made" (p. 4). Kant (1989) as quoted by Von Glasersfeld, 2001) echoes that "human reason can grasp only what she herself has produced according to her own design (p. 4)".

Tyler (1993), as cited in Duit, 1996) asserts that:

> The common constructivist core is a view of human knowledge as a process of personal cognitive construction, or invention, undertaken by the individual who is trying for whatever purpose to make sense of her social or nature environment. (p. 41).

Constructivists do not see learners as passive receiver but as active constructors of knowledge. Constructivism enables learners to participate in the production of knowledge in the classroom (Flynn, Vermette, Mesibov, & Smith, 2004). Learners learn through interaction with problems and others, and develop what Vygotsk calls the tools of intellectual interpretations, which consequently become an essential part of the learning process (Vygotsky, 1978).

Key to interacting with mathematical problems, whether in a collaborative learning environment or not, is the use of visual processes as these facilitate meaningful deconstruction of mathematical concepts and ideas through diagrams and other illustrations. The notion of this type of interaction thus aligns well with constructivist ideas that learning takes place through active construction of knowledge, in this case through the use of visualisation processes.

Bruner (1966) believes that young children learn through manipulation and action (**enactive representation**), older children learn through perceptual organization and imagery (**iconic representation**), and adolescents learn through the use of language and symbolic thought (**symbolic representation**). Clements (1999) suggests that all three types of representation (enactive, iconic and symbolic representations) should be used in parallel to facilitate student learning. He further states that when students make connections between concrete, pictorial, and symbolic representations, their learning is enhanced and increased.

This study advocates that in a mathematics class learners should use different visualisation processes such as objects, actions, pictures, symbols, and words to represent and deconstruct mathematical ideas. These could be likened to Bruner's three types of representations, with objects and actions being enactive, pictures being **iconic**, and symbols and words being **symbolic**.

Significance of the study

Students use visualisation processes as tools to support their mathematical understandings. For teachers, the use of visuals in the lessons presentations can influence students' knowledge of representations. This, in turn, is related to students' mathematical achievement. Therefore, it is important to examine the nature of mathematical visuals and their use in Mathematics.

The proposed research could provide insightful information on algebraic problem solving that could be useful to teachers, learners and curriculum developers. Further, it could also enable teachers to understand the significance of visualisation and the need to be able to translate complicated mathematical ideas into representations that students can understand. For curriculum developers this study could be informative in foregrounding the need to use appropriate visualisations in class in general and for algebraic problem solving in particular. This understanding can lead to the improvement of the curriculum.

Research goals and research questions

The aim of the study is to analyse selected learners' visualisation processes in solving algebraic problems and find out how they are used. The two research questions that frame the study are:

- What problem solving strategies using visualization processes are employed by selected Grade 11 and 12 learners when solving algebraic problems?
- How do these Grade 11 and 12 learners use the identified problem solving processes to solve algebraic problems?

Methodology

Orientation

According to Bertram & Christiansen (2014), interpretivist researchers "aim to understand how people make sense of their worlds?" (p. 26). Cohen, Manion and Morrison (2011) explain that:

> …the central endeavour in the context of the interpretive paradigm is to understand the subjective world of human experience. To retain the integrity of the phenomena being investigated, efforts are made to get inside the person and to understand from within. The imposition of external form and structure is resisted, since this reflects the viewpoint of the observer as opposed to that of the actor directly involved. (p. 17)

When solving algebraic problems, students use multiple visualisation processes that may lead to the solution. Some might visualise the problem externally for example by drawing pictures, graphs and diagrams on paper and some might visualise the problem internally by making visual representations in the mind. The students' visual diagrams can only be understood when they interpret them.

My research project is therefore oriented within interpretive paradigm as I wish to gain a deeper understanding of how learners solve algebraic problems with respect to the visualisation processes that they use. My approach will be qualitative.

Research Method

The site for the proposed research consists of one school only. A small sample of 3 grade 11 and 3 grade 12 learners will be used. Thus, a qualitative case study would be the most suitable method for my research. According to (Rule & John, as cited in Bertram, & Christiansen, 2014), "a case study is a systematic and in-depth study of one particular case in its context" (p. 42). The case will be a group of high ability grade 11 and 12 learners. The unity of analysis will be the responses of the learners to selected algebraic tasks i.e. the visualisation processes that they use and how they used them.

To collect data for this study, I am going to use different research tools such as, interviews, learners' work (responses to the Visualisation Algebraic Tasks-AVT) and observation. The interviews will be done to find out and interpret the visualisation processes that the learners employed. Learners' work will be used to analyse the visualisation processes used to solve the AVT tasks. Observations will be used to further identify the visualisation processes used and how they were used. A template will be designed to assist in identifying and categorising the visualisation processes.

Research design

My research will go through three different phases. During the **first phase**, I will identify the site and select the participants. I am also going to refine the Algebraic Visualisation Tasks (AVT) activity sheet and then test the validity of this instrument by doing a pilot with 4 grade 11 and 12 learners. The AVT will consist of ten algebraic problems.

The **second phase** will be the implementation of the AVT with the selected grade 11 and 12 learners. In this phase learners are going to work on the AVT and they will be observed and interviewed while tackling the individual AVT items. I intend to interact with each participant as he/she "talks through" each of the items whilst solving them. I will video record these interactions to capture what the participants say and draw. Pictures of the learners' work will also be collected.

In phase three I will analyse the video recordings and identify the visualisation strategies that the participants used. To address research question two, a detailed analyses of the videotapes will be done guided by the literature review. This will help me to note what learners go through when solving algebraic problems and how they use the visualisation processes to solve the problems.

Participants

My research participants are 6 high ability grade 11 & 12 learners (3 learners from grade 11 and 3 from grade 12). I wish to have a balanced gender mix. I am seeking the participation of high achievers because I want learners who are proficient algebraic problem solvers and who are likely to use different visualisation processes. Further I wish my participants to be good communicators and able to articulate their thoughts and their learning processes. So the learners will be selected based on their capability and also willingness to participate in the study. My selection strategy is thus purposeful. According to Bertram & Christiansen (2014), purposeful sampling is when the researcher makes specific choices about which people, groups and objects to include in the sample. Furthermore, my aim is to collect rich in-depth data. Hence my focus is only on one school. I intend to use the school that I teach at because I am familiar with the learners and the ethos of the school. I have a good relationship with my colleagues and other stakeholders at the school. We have a good understanding of mutual trust and respect.

Data collection Tools

Algebraic visualisation tasks (AVT)

The AVT consists of 10 different algebraic problems that will be set and given to the participants. They will be encouraged to solve the problems using visualisation processes such as graphs, drawings and sketches. I will sit with the learners whilst they engage with the AVT items and they will be required to give reasons of their choice of drawings/diagrams used, as well as to explain how such diagrams assisted them in solving the given algebraic problems. They will also be asked to explain why they opted to use these diagrams/drawings over any others. Learners will be encouraged to talk and explain their problem-solving processes while tackling the algebraic problems. This will give me (as a researcher) a deep insight of their thoughts and the visualisation processes they go through as they attempt to solve the AVT items. Learners will be videotaped while working on the problems and engaging with me.

Below are examples of five algebraic tasks that will form the AVT. They are adopted from the Namibia senior secondary certificate mathematics higher level examination paper 1 of 2012.

1. One side of a rectangle is 3 cm shorter than the other side. If we increase the length of each side by 1 cm, then the area of the rectangle will increase by 18 cm^2. Find the lengths of all sides.
2. Hafoletu is putting up a tent for a family reunion. The tent measures 16 m by 5 m. Each 4-m section of tent needs a post except the sides that are 5 m long. How many posts will he need?
3. A moving company is hired to take 578 clay pots to a florist shop. The florist will pay the moving company a N$200 fee, plus N$1 for every pot that is delivered safely. The moving company must pay the florist N$4 each for any pots that are lost or broken. If two pots are lost, four pots are broken, and the rest are delivered safely, how much should the moving company is paid?
4. A box has a volume of 480 cm^3, a breadth of 4 cm, a length of ***x*** cm and a height of *h* cm. Express the height ***h*** in terms of ***x*** and show that the total surface area of the box, (*A* $cm^{2)}$, is given by A= $8x + \frac{960}{x} + 240$
5. **f** is a quadratic function whose graph has a vertex at the point (-3 , 2) and has a y-intercept at the point (0 , -16). Find the x-intercepts of the graph of **f**.

According to our regional scheme of work, algebra is supposed to be covered in the first term of grade 11. By the time this research will be conducted algebra would have already been taught. Therefore, all algebra concepts in the grade 11 and 12 curriculum can be included in the AVT. The learners will be provided with ample blank paper to scribble on whilst solving the 10 tasks.

Interviews

The main source of data for this research will be the transcripts of the solution strategies, interviews and conversations I have with the participants. According to Bertram and Christiansen (2014) interviews are used in working towards the aim of “exploring and describing peoples’ perceptions and understanding that might be unique to them” (p. 82). They further state that interpretivist research uses the interview method extensively, since it allows the researcher to ask probing and clarifying questions and to discuss research participants’ understanding with them. For this study, the interviews with the individual learners will take the form of a conversation whereby the learners will be asked to verbally explain how they used their visualisation processes to solve each of the AVT tasks. According to Cohen and Manion (1994) the unstructured interview is an open- ended approach to interviewing in which questions flow from the immediate context.

During these conversations learners will be asked to explain their drawings or any visualisation processes they use. Their visual representations may not all be self-evident (Steinbring, 2005). I may have to ask for clarifications and elaborations from the learners as they engage with these representations.

Observation

In conjunction with conversing with the participants as articulated above I will also video record these interactions and use the video recordings as supplementary data. I will use the video data to corroborate the conversation data and subject it to a similar analysis process – see data analysis below. Simpson & Tuson (2003, p. 48) encourage that “if we are dealing with people, video recording can be a great help as it allows the same observation to be reviewed many times, with each viewing having the potential to elicit additional information”.

Data analysis

Algebraic visualisation tasks (responses)

In order to identify, classify and code the visualisation processes that the participants used I will analyse the interview/conversation and video data concurrently with the aid of the analytic templates which I have constructed from selected key resources in the literature See Table 1 and Table 2. The templates are works in progress and still need refining.

In the analysis of the data I will use table 2 to categorise the visual processes used for each task of the AVT that each participant solved. From the learners’ activities the processes used will be identified as either **external** or **internal** visualisation (Kashefi, Alias, Kahar, Buhari & Zakaria, 2015). According to Chiappini and Bottino (2010) external representations “are two or three-dimensional representations of some aspects of a mathematical structure” (p.1). Goldin and Kaput (1994, p. 400) define external representation as “physically embodied, observable configurations such as words, graphs, pictures, equations, or computer micro worlds”. The internal visualisation is defined by Goldin (2002) as “the internal systems of representation that are created within a person’s mind and used to assign mathematical meaning”. (p. 178). These are manifested verbally during my conversation with each participant.

I will further analyse if the representation is a **descriptive representation** or **deductive** representation. Schnotz & Bannert (2003,) define descriptive representations as representations that are spoken words or written texts such as mathematical equations and logical expressions. A depictive representation consists of iconic signs such as pictures, sketches, or drawn models (p. 143). Moreover, Schnotz et al... (2003) articulates that “if the descriptive representation of a function is made by the term 2x + y = 0 then a corresponding depictive can be a straight line graph in a Cartesian plane that passes through the origin” (p. 32). In other words 2x + y = 0 is a descriptive representation and the straight line diagram is the depictive representation.

Also, I will look at why and how participants used certain visualisation processes to solve the algebraic problems. According to Carney et.al (2002), there are multiple functions that pictures serve. Pictures can be used as **fundamental tools;** they can also be used **to illustrate, to simplify, to organize ideas and as starting point.**

Table 1 is my visualisation analysis template that will be used to analyse the visualisation processes of each participant for each AVT task.

Observable indicator	**coding**		
	1	**2**	**3**
External visualisation *These are observable shapes such as diagrams, graphs and pictures. It also includes representations such as equations and words.*			
Internal visualisation *These are representations that are drawn in the mind and are imagined. These are only made explicit verbally and manifest themselves in conversation.*			
Descriptive representation *This is similar to the internal visualisation indictor above and refers to those representations that are spoken words or written texts such as mathematical equations and other expressions*.			
Depictive representation *These are iconic signs such as pictures, sketches, or drawn models* and they are similar to the external indicators above.			
Fundamental tool *This is when an overall visualisation representation is used throughout the problem.*			
Illustrative tool *This is when a visual representation is used to illustrates the problem in a familiar way.*			
Simplification tool *This is when a visual representation is used to make the problem easier.*			
Organizational tool *This is when the visual representation is used to deconstruct the problem into smaller components.*			
As a starting point *This is when the visual representation is used as an initial starting point that is developed further.*			

Table 1: Analytic Template A - Categories of visualisation processes.

Table 2 below illustrates the coding categories that I will apply to the observable indicators above.

Coding template
1- very strong evidence When the visualisation process used is indicated clearly, **it is visible** and **observable**. There is strong evidence that the visualisation process was used. **2- mediocre evidence** This is when there is **mediocre evidence** that the visualisation process was used. The representations are **tentative**. The representations are **not clear** – they are mere scribbles and very rough sketches. **3-weak evidence** This is when the student claims to have used a certain visualisation process but there is no evidence that it was really used. No picture, drawing, graph or table drawn or the student cannot explain the picture that had been drawn in mind.

Table 2. Analytic Template B - Coding visualisation processes used in each AVT task.

Interviews and the video recordings

As already indicated I will use the above templates as I am engaging with learners as they solve each task of the AVT and I will also use the same templates again as I analyse the video recordings. This analysis of the video recording will ensure that I capture all the visualisation processes evident for each learner for each task.

In terms of the second research question, how the learners use the visualisation strategies, the verbal responses to each word problem will be transcribed and read several times to establish a good sense of the data.

Table 3: Summary of data generation and analysis phases

Phases	instrument	Purpose	Data	Analyses
Phase 1: Selection of the site and the participants . Seeking consents Development and piloting of AVT	N/A N/A AVT	To gain permission to collect data and get potential participants for the research. To gain acceptance and consent from intended participants. To make any improvements and adjustments to the AVT	N/A N/A N/A	N/A N/A N/A
Phase 2: Administration of AVT	AVT	To generate data	The verbal, written and drawn responses to the questions in the AVT	N/A
	observations Interviews video recordings	To generate data	The verbal, written and drawn responses to the questions in the AVT.	N/A
Phase 3 Data analysis	Own designed data analysis tool	To answer the research questions. To analyse the findings To suggest conclusion.	The verbal, written and drawn responses to the questions in the AVT.	Qualitative analyses

VALIDITY

Researchers need to ensure that their data collection methods and data are as valid or trustworthy as possible (Bertram & Christiansen, 2014, p.176). Gregory (1992, p.117) defines validity as "the extent to which a test [in this case the VAT] measures what it claims to measure". I will have an appropriately sized sample of six learners (three in grade 11 and three in grade 12). These learners will be selected according to their capability in mathematics and their communication skills to ensure they will be able to solve the AVT using visualisation strategies and talk through during the AVT. Before administering my AVT to the research participants I will pilot the AVT to see the validity of my instrument and make amendments. The participants are my learners, so we are familiar with each other. This helps them to be free to express their ideas and explain their processes. Furthermore, all the interviews and observation will be video recorded.

Ethics

See attached document!

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RU FACULTY OF EDUCATION: ETHICAL APPROVAL APPLICATION

IMPORTANT: The following form needs to be completed by the researcher and submitted with their research proposal to the Education Higher Degrees Committee. The details to which this form relates should also be evident in the text of the proposal.

GENERAL PARTICULARS

MEd (Half thesis)		**MEd (Full thesis)**	**X**	**PhD**		**Other: Please specify**	

TITLE OF RESEARCH: An analysis of visualization processes used by selected Grade 11 and 12 learners when solving algebraic problems.

DEPARTMENT/INSTITUTE : Education Department

DATE : October 2015

RESEARCHER : Joseane Josef
SUPERVISOR : Pro Marc Schäfer

ETHICS

NB: You must read the Faculty of Education Ethics Guideline ***prior*** to completing this form. Please indicate below how your research supports the indicated ethical principle:

Respect and dignity

The study is intended to be done at the school that I teach at because am familiar with the learners and the ethos of the school. I have a good relationship with my colleagues and other stakeholders at the school. We have a good understanding of mutual trust and respect. Moreover, participants will be ensured of their anonymity and be informed that this will be respected and retained throughout the study. It will also be made clear to them that they have the right to withdraw from the study. Participants will also be informed that, all sessions will be conducted after school to ensure they will not miss out on their normal classes.

Transparency and honesty

The permission in the form of written consent will be obtained from the Department of Education, the school principal of the school where the research will be conducted. The participants as well as parents/guardians of the learners who will agree to participate in the research will also be issued with a consent form that they will be required to sign and return agreeing to be part of the study. Moreover, when interviews are transcribed, interview transcripts will be shared with participants to ensure that data was appropriately and ethically collected and reported.

Accountability and responsibility

My participants will be interviewed and the fact that they are my learners mighty affects the way of answering the questions. Before I start with interviews, I will make it clear to the participants that despite me being their teacher, they must take me just as an ordinary researcher and answer my questions honestly and truthfully.

Integrity, academic professionalism and researcher positionality

The Study will follow a qualitative case study due to the fact that the site for the intervention consists only one school and a small sample of 6 grade 11 and 12 learners. To ensure validity numerous sources of data will be used such as interviews, observations and learners' work. All interactions with the learners will be video recorded and pictures of their work will also be collected. I am also going to declare that the study is my original work and I will acknowledge by referencing ideas of others.

Signature (researcher): Joseane Josef

Signature (supervisor): Marc Schäfer

Date: 15 October 2015

Place: NIED, Okahandja